Microeconomics III
Fall 2021

Assignment 1 : Solution Guide

To pass, you need to get at least half of the points. All four questions are of equal value.

1. Consider the following normal form game:

	a	b	c
A	$1, 1$	$3, 4$	$6, 0$
B	$4, 3$	$1, 1$	$2, 2$
C	$0, 6$	$2, 2$	$5, 5$

(a) Can you eliminate any strategies using IESDS?

Yes. For player 1 strategy C is strictly dominated by strategy A. Once C has been eliminated from the game, strategy c for player 2 is now strictly dominated by a. The reduced game is now:

	a	b
A	$1, 1$	$3, 4$
B	$4, 3$	$1, 1$

(b) Find all pure strategy Nash equilibria of the game.

By highlighting the best responses, we see there are two pure strategy Nash equilibria. The Nash equilibria are given as $PSNE = \{(B, a), (A, b)\}$

	a	b
A	1, 1	3, 4
B	4, 3	1, 1

(c) Is there a mixed strategy Nash equilibrium with full support, i.e. is there a mixed strategy NE where all pure strategies are played with positive probability? Why or why not?

When looking at the full 3×3 game, there is no mixed strategy with full support as strategies C and c are both played with probability 0, as they are strictly dominated. Strictly dominated strategies will never be part of a mixed strategy.

(d) Find a mixed strategy equilibrium where both players mix between two pure strategies.

We can assign probability p to strategy A, probability $1 - p$ to strategy B, probability q to strategy a and probability $1 - q$ to strategy b.

P1 is indifferent between A and B when:

$$q + 3 - 3q = 4q + 1 - q$$
$$2 = 5q$$
$$q = 2/5$$

P2 is indifferent between a and b when:

$$p + 3 - 3p = 4p + 1 - p$$
$$2 = 5p$$
$$p = 2/5$$

MSNE $= (p, q) = \{\frac{2}{5}, \frac{2}{5}\}$

2. For each statement below, state whether it is TRUE or FALSE and briefly motivate your answer. Informal discussion is enough (2-3 sentences each).

   (a) The reason that players cannot achieve a good outcome in the prisoner's dilemma is that they cannot communicate. FALSE. Even when the players can communicate with each other and agree to coordinate on "stay silent", there is still incentive for both players to deviate to "confess". This occurs because there is no punishment for a player who goes back on their word.

   (b) Iterated Elimination of Strictly Dominated Strategies never eliminates a Nash Equilibrium. TRUE

   (c) Consider a n-player normal form game and assume that both actions a_i and a_i' are best responses for Player i to strategy profile s_{-i}. This implies that $pu_i(a_i, s_{-i}) + (1-p)u_i(a_i', s_{-i})$ is independent of parameter p.

   TRUE. If both a_i and a_i' maximize Player i's payoff, they must give the same payoff. Hence, any convex combination gives the same payoff too. (This observation is useful: among other things we implicitly use it when finding the mixed NE by using indifference.)

3. (More difficult) Two firms choose simultaneously their levels of production, q_1 and q_2. Firm 2 already has produced $\bar{q} \in [0, 1/2]$ and will have to sell that quantity no matter what. $q_2 \geq 0$ is what Firm 2 can produce additionally so that its total supply is $\bar{q} + q_2$. Products are identical and the inverse demand is $P(q_1, q_2, \bar{q}) = 1 - q_1 - q_2 - \bar{q}$. The marginal cost of production is 0 for both firms.

   (a) Write the game in normal form.

   $(\{Firm1, Firm2\}, \mathbf{R}_+ \times \mathbf{R}_+, (u_1(q_1, q_2) = P(q_1, q_2, \bar{q})q_1, u_2(q_1, q_2) = P(q_1, q_2, \bar{q})(q_2 + \bar{q}))$ where

   - $\{Firm1, Firm2\}$ is the set of players,
   - $\mathbf{R}_+$ is the set of pure strategies/actions,
   - u_i is the payoff function for Firm i.

   (b) For each $\bar{q} \in [0, 1/2]$, find the Nash equilibrium of the game.

   Let's start with Firm 1:

$$u_1(q_1, q_2; \bar{q}) = (1 - q_1 - q_2 - \bar{q})q_1.$$

   The first order condition:

$$1 - q_2 - \bar{q} - 2q_1 = 0 \iff B_1(q_2; \bar{q}) = \max\{\frac{1 - q_2 - \bar{q}}{2}, 0\}.$$

   Similarly for Firm 2:

$$u_2(q_1, q_2; \bar{q}) = (1 - q_1 - q_2 - \bar{q})(q_2 + \bar{q}).$$

   The first order condition:

$$1 - q_1 - 2q_2 - 2\bar{q} = 0 \iff B_2(q_1; \bar{q}) = \max\{\frac{1 - q_1 - 2\bar{q}}{2}, 0\}.$$

   Plugging in $B_1(q_2; \bar{q})$ to the first order condition of Firm 2 yields:

$$1 - \frac{1 - q_2 - \bar{q}}{2} - 2q_2 - 2\bar{q} = 0 \iff q_2 = \frac{1}{3} - \bar{q}.$$

   The Nash equilibrium is $q_2^* = \max\{\frac{1}{3} - \bar{q}, 0\}$ and $q_1^* = \frac{1 - \max\{1/3, \bar{q}\}}{2} = \min\{\frac{1}{3}, \frac{1-\bar{q}}{2}\}$.

(c) Based on your answer to part (b), is Firm 2 better off when $\bar{q}$ is low (close to 0) or when it is high (close to 1/2)? Can you explain why (write 1-2 sentences)?

If $\bar{q} \leq 1/3$, it does not affect total amount of sales: Firm 1 sells and produces 1/3 and Firm 2 sells 1/3 and hence produces $q_2 = 1/3 - \bar{q}$. In this case, the storage does not affect either firm's profits.

If $\bar{q} > 1/3$, it affects total sales and the division of sales between the two firms: Firm 1 sells and produces $(1 - \bar{q})/2$ and Firm 2 does not produce more, $q_2 = 0$, and sells $\bar{q}$. In this case, the storage makes Firm 2 better off and Firm 1 worse off. To see the former, calculate the payoff for Firm 2 as a function of $\bar{q}$:

$$u_2^*(\bar{q}) = u_2(q_1^*, q_2^*; \bar{q}) = (1 - (1 - \bar{q})/2 - \bar{q})\bar{q} = 1/2(1 - \bar{q})\bar{q}$$

$u_2^{*\prime}(\bar{q}) = 1/2(1 - 2\bar{q}) > 0$ for all $\bar{q} < 1/2$.

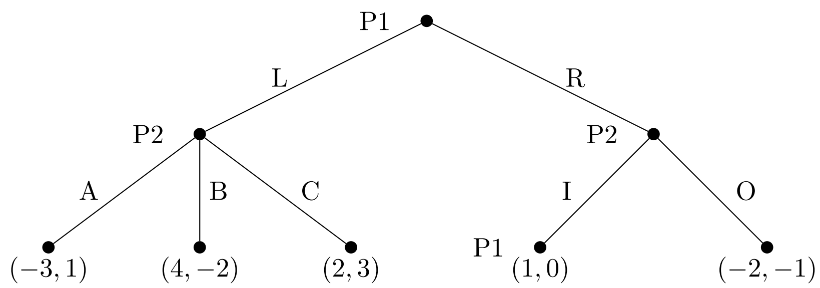
Large storage helps Firm 2 because it makes Firm 1 to produce less. In fact, the storage enables that Firm 2 can act like the Stackelberg leader: $q = 1/2$ would be the quantity chosen by the first mover in the Stackelberg game.

4. Consider the extensive form games below with two players (P1 and P2).

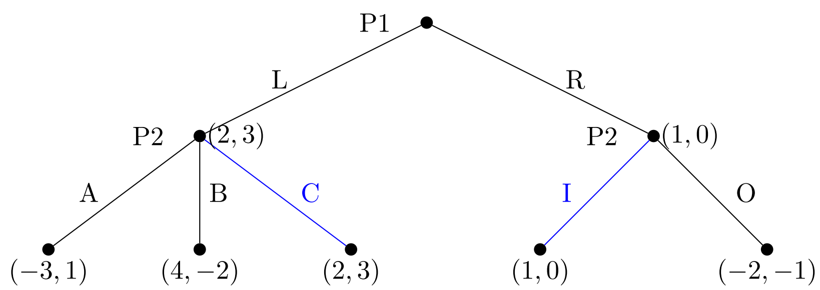
(a) What are the pure strategies for each player in Game 1?

Player 1 has one decision node in which he can play either L or R. Combining these choices gives $2^1 = 2$ different strategies. S_1 can be written as $S_1 = \{L, R\}$. Player 2 has two decision nodes, one in which he can play A, B or C and one in which he can play either I or O. Combining these choices gives $3^1 \times 2^1 = 6$ different strategies. S_2 can be written as $S_2 = \{AI, AO, BI, BO, CI, CO\}$.

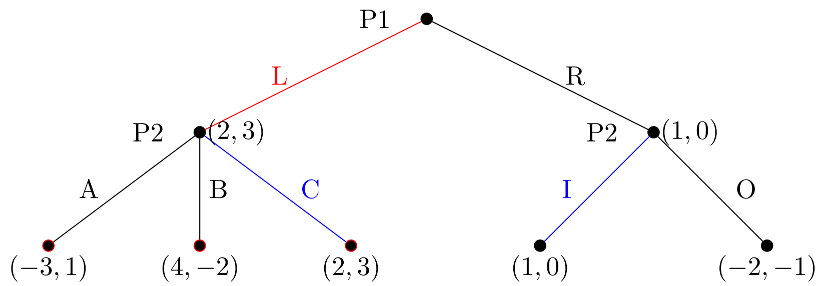
(b) Solve Game 1 using backward induction. Game 1:



When applying backwards induction, we first look at the last possible nodes. In this case the nodes of player 2. For player 2's left node, their best response is to play C, as $3 > 2 > -2$. For player 2's right node their best response is to play I as $0 > -1$. We show this in the game tree by highlighting the paths players 2 would choose, and adding the payoffs from those choices to their left and right nodes respectively. These nodes now show the payoff that player 1 will get from making a choice that leads to that node.



Once we have player 2's best responses, we can then follow the same process for player 1's single decision node. Player 1's best response at their decision node is to play L as $2 > 1$.



The BI outcome is that Player 1 plays first L and then Player 2 plays C. The BI strategy profile is (L, CI).

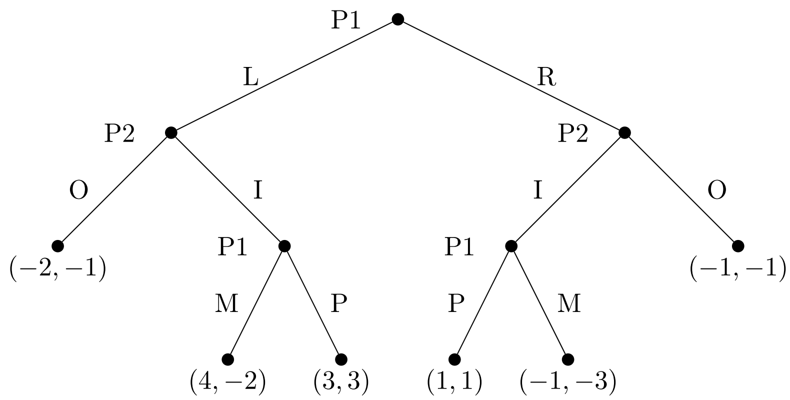
(c) What are the pure strategies for each player in Game 2?

When writing the game normal form, we remove all information about turns. Player 1 has three nodes with two options each, L or R, M or P in the "L" side and M or P in the "R" side. Combining these choices gives $2^3 = 8$ different strategies. Player 2 has two nodes with two possible actions each, giving $2^2 = 4$ different strategies. Remember that players pure strategies must include all possible actions, even those off the equilibrium path.

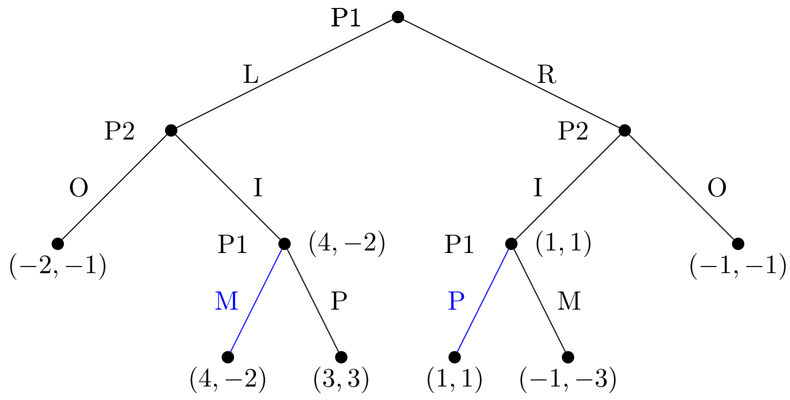
Eventually, the pure strategies can be written as
$S_1 = \{LMP, LMM, LPP, LPM, RMP, RMM, RPP, RPM\}$
$S_2 = \{OI, OO, II, IO\}$

(d) Solve Game 2 using backward induction.

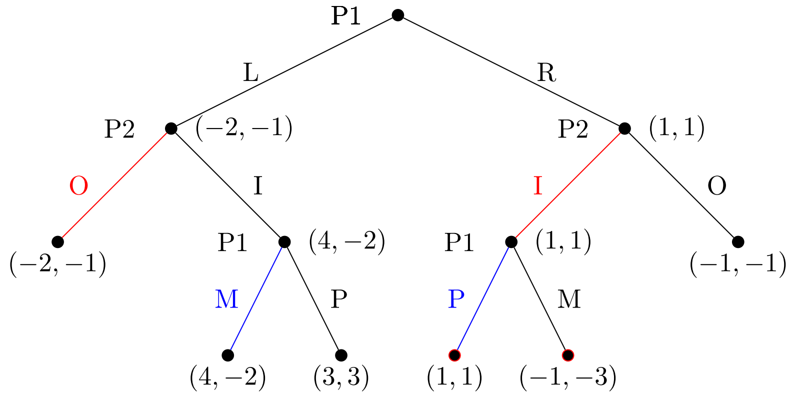
Game 2:



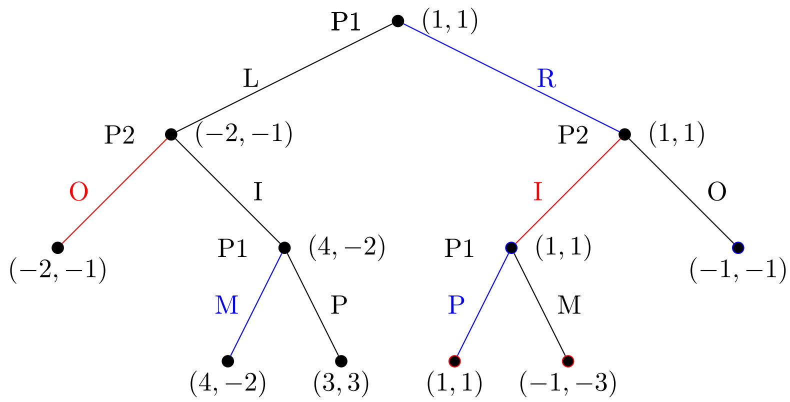
When applying backwards induction, we first look at the last possible nodes. In this case the LI node and the RI node (the nodes that come after playing L,I and R,I respectively). For the LI node, player 1 would play M, since that choice offers a payout of 4, which is higher than the payout of 3, which he would get from playing P. For the RI node, player 1 would play P. We show this in the game tree by highlighting the paths players 1 would choose, and adding the payoffs from those choices to the LI and RI nodes respectively. These payoffs now show the payoff that player 2 will get from making a choice that leads to that node.



Once we have player 2's best responses, we can then follow the same process for the L and R node, to find that player 2 would play O given L, and I given R.



The final piece is to look at Player 1's choices, which now has been reduced to getting -1 from playing L, and 1 from playing R, so player 1 will play R. Hence, Backward induction gives path 'RIP' and payoffs $(1, 1)$.



The BI outcome is that Player 1 first plays R, player 2 then plays I and lastly Player 1 plays P again. The BI strategy profile is (RMP, OI).