Microeconomics III
Fall 2021

Assignment 4 : Solution Guide

1. Tragedy of the commons.
   (a) Give a real-life example of a situation of the tragedy of the commons (do not use the examples in the textbook or lecture notes).
   (b) Write your example (or a simplified version of it) as a normal form game.
   (c) Are any strategies strictly dominated? Which strategies are rationalizable? (i.e. survive IESDS)
   (d) Find all pure and mixed NE of the game.
   (e) Interpret your results.

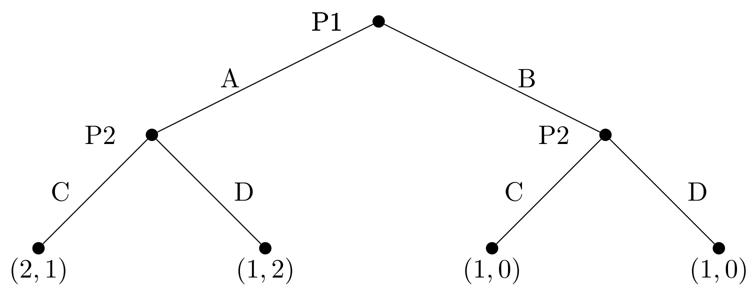
   Comment: this is a demanding – but also very educating – exercise so grading should be lenient

2. Consider the dynamic game with the normal form given by the bi-matrix below:

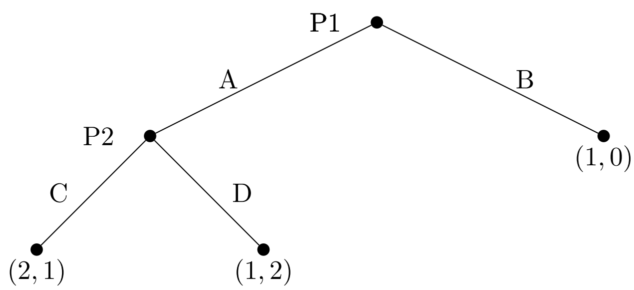
		Player 2 C	Player 2 D
Player 1	A	$2, 1$	$1, 2$
	B	$1, 0$	$1, 0$

   (a) Let Player 1 move first. Draw the corresponding perfect information game tree (you may leave out degenerate action choices that do not affect any player's payoffs). What are the pure strategy SPE? What are the maximum and minimum SPE payoffs for each player? (Hint. there are two different SPE payoffs.)

   Solution:
   The corresponding perfect information game tree:
   Game tree 2a.1:



   Following player one chooses B, players twos choice does not affect the payoffs, and thus can be considered degenerate. Instead, you could choose to model the game as B being an outside option for player 1.
   Alternatively, **Game tree 2a.2** (It is enough to draw either):



We will be using Game tree 2a.1 to find the SPEa.

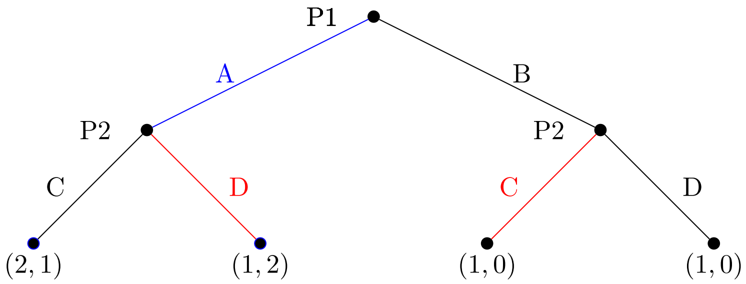
When doing this we encounter multiple instances where a player is indifferent between two actions, and thus both actions is a best response. Whenever two options are both considered a best response in a dynamic game tree, we split up the game tree, and analyse each of those variations on their own, leading to multiple SPEa.

In this game, player 2 is indifferent when choosing between C and D following B, meaning we have to analyse two variations. In both of those variations, for the subgame perfect choices of player 2, player 1 is indifferent between A and B. This leads to another split, meaning we now have $2 * 2 = 4$ different variations to analyse.

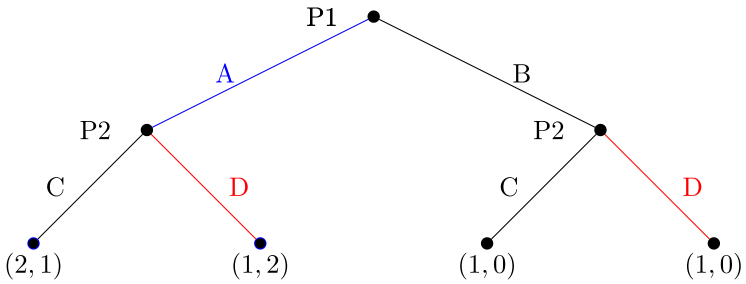
The resulting SPEa are: (A,DC), (A,DD), (B,DC), (B,DD). The set of pure strategy SPE payoffs is: $\{(1,2),(1,0)\}$. The maximum SPE payoff for player 1 is 1, for all the SPE. Player two gets his maximum payoff for the two SPEa: (A,DC), (A,DD).

The four different SPEa are illustrated below:

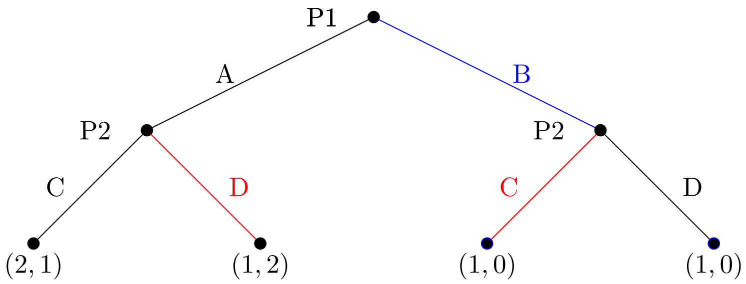
SPE (A,DC)



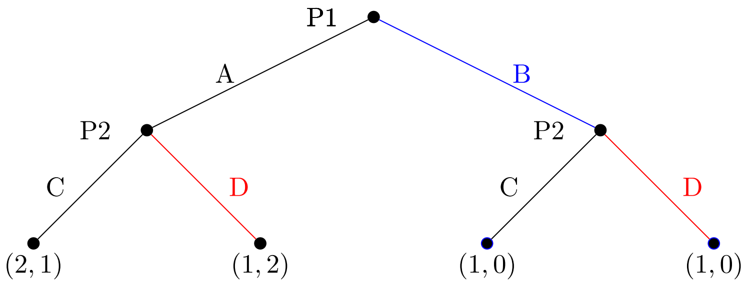
SPE (A,DD)



SPE (B,DC)



SPE (B,DD)



If using Game tree 2a.2, there are two SPEa: (A,D), (B,D).

(b) Let Player 2 move first. Draw the corresponding perfect information game tree. What are the pure strategy SPE? What are the maximum and minimum SPE payoffs for each player? (Hint. there are two different SPE payoffs.)

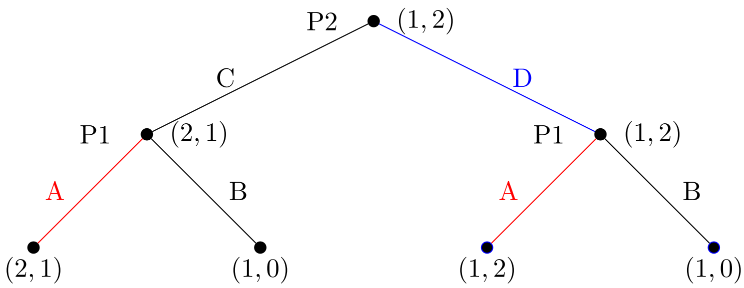
Solution:

This game is solved in a similar manor to 2a. Player 1 is indifferent in the note following D, hence two SPEa.

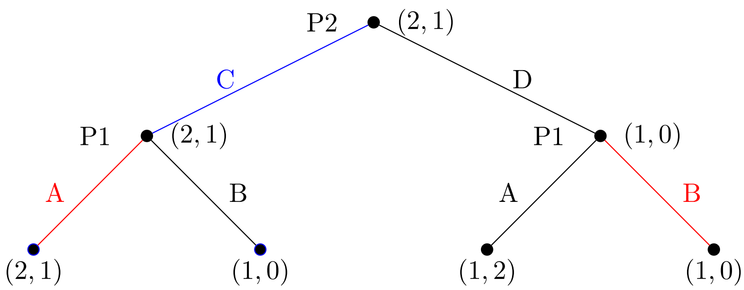
SPE: (AA,D),(AB,C)

The set of pure strategy SPE payoffs is: $\{(1,2),(2,1)\}$. Player 1 gets the highest payoff, with a payoff of 2, from (AB,C). Player 2 gets the highest payoff, with a payoff of 2, from (AA,D)

SPE (AA,D)



SPE (AB,C)



(c) Discuss whether there is a first-mover or last-mover advantage in the game. Does your answer depend on which equilibrium is played? (Hint. there is no one correct way of answering this question. You may, for instance, want to compare the minimum and maximum payoffs respectively or the payoffs arising only in (a) to those arising only in (b). You can also compare your results with the static game...)

Solution:

If we compare the entire equilibrium sets, both players are better off (technically in weak set order) if player 2 moves first. Player 1 is better off in b) if we compare the maximum equilibrium payoffs and Player 2 is better off if we compare minimum equilibrium payoffs.

One could say that Player 1 has (weak) last-mover advantage and Player 2 has (weak) second-mover advantage, and therefore their preferences on the order of moves are aligned.

3. **Signaling.** Imagine that there are several firms that sell used cars. Most of these firms sell cars of poor quality, which might fall apart at high speeds or catch fire during hot weather. There are a few firms who sell good cars. All consumers would prefer to buy good cars (and would be willing to pay a higher price), but it is impossible to tell the quality of a used car, and therefore the firms that sell good cars are not doing well.

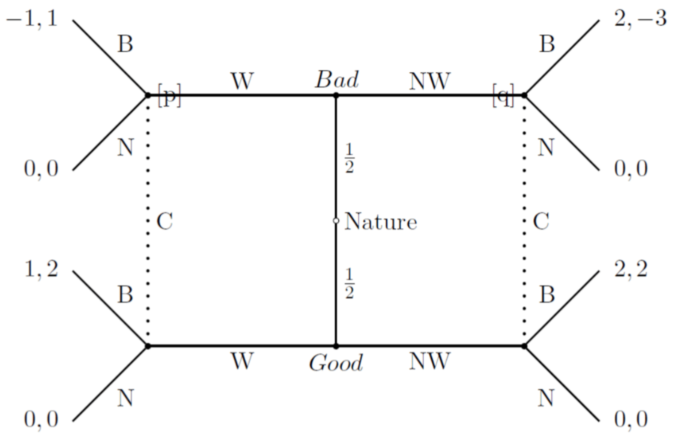
   They have tried to convince consumers that their cars are good by putting stickers on them that say "good car", but consumers did not seem to care. Now an employee of a firm that sells good cars has read a book about dynamic games with incomplete information and tells her boss that they should offer a warranty: if somebody buys a car from them and it breaks down, they will get a new one.

   (a) Explain briefly (1 sentence) why the stickers didn't work.

   Solution: Sellers of bad cars can also put these stickers on their cars without any cost: this a cheap talk game where the sender always prefers the same action from the receiver and hence a pooling equilibrium is the unique PBE.

   (b) We can model the game as follows: There are just as many good cars as bad cars, and for every car the seller can choose whether to give a warranty (W) or not (NW). Giving a warranty for a bad car is more costly than for a good car (because the probability that it needs to be replaced is higher). In each case, the consumer C can decide whether to buy (B) or not (N) (if he doesn't buy, nobody gets anything). The extensive form is below.

   Find a Perfect Bayesian Equilibrium (PBE) in which only good cars get warranty. Remember to check Requirements 1-3.



   Solution: Consider the separating PBE where sellers of bad cars don't give warranty, but sellers of good cars do. Go through the signaling requirements:
   SR3: The beliefs of the consumer C that are consistent with this separating strategy are that it's a good car if the sellers gives warranty and a bad car if not, i.e.

$$\mu(Bad|W) = p = 0 \text{ and } \mu(Bad|NW) = q = 1$$

   SR2R: Given these beliefs, the consumer buys a car with a warranty but does not buy a car without a warranty as:

$$\mathbb{E}[u_C(W,B)|p=0] = 2 > 0 = \mathbb{E}[u_C(W,N)|p=0]$$
$$\mathbb{E}[u_C(NW,N)|q=1] = 0 > -3 = \mathbb{E}[u_C(NW,B)|q=1]$$

SR2S: Sellers of good cars nor sellers of bad cars want to deviate as:

$$u_S(NW, N|Bad) = 0 > -1 = u_S(W, B|Bad)$$
$$u_S(W, N|Good) = 1 > 0 = u_S(NW, N|Good)$$

As there is no incentive to deviate, $PBE = \{(NW, W), (B, N), p = 0, q = 1\}$

EXTRA: There are no other PBE in pure strategies. There cannot be a pooling equilibrium in which both cars get warranty, because then the consumer will buy after seeing a warranty and the Bad type has an incentive to deviate. There can neither be a pooling equilibrium in which both cars don't get warranty: Then the consumer would not buy after seeing no warranty, and the Good type can gain from deviating (since there is no belief p such that the consumer doesn't buy after observing W). Finally, there can be no PBE in which only the bad cars get warranty, since the consumer will then buy a car if it doesn't have a warranty (believing it to be a good car), and the Bad type has an incentive to deviate.

(c) Why does it make sense for the firm to give a warranty? Explain the intuition briefly (3-5 sentences).

Solution: Giving warranty is a costly signal. It does not pay off for the sellers of bad cars to give warranty. Therefore, the consumer knows that if a car has warranty, that car is a good car. The sellers of bad cars have no incentive to try to imitate the sellers of good cars, because the warranty is so costly for a bad car that it eliminates all advantages from looking like a good car.