Assignment #2: Relationship Between GDP and Consumer Confidence

1 Introduction

This paper examines the relationship between gross domestic product (GDP) and consumer confidence (CC) by applying Granger causality to determine to the causal direction, so it can be established whether GDP induces changes in CC or the reverse. We use a vector autoregressive (VAR) model to analyze the connection between the two economic variables. By using a general-to-specific (GETS) method a VAR(7)-model is chosen. It is determined that CC Granger causes GDP, where both a short-run and long-run effect are determined.

2 Description of data

This paper uses data from Germany and examines the relationship between the GDP and CC. The data has been downloaded from FRED database maintained by the Federal Reserve Bank of St. Louis.
We define the variables:

- $y_t = \log(GDP_t)$ and $\Delta y_t = y_t - y_{t-1}$, where Δy_t is the first difference of the logarithm of GDP for Germany in millions of chained 2010 Euros and seasonally adjusted.

- $c_t = \log(CC_t)$ and $\Delta c_t = c_t - c_{t-1}$, where Δc_t is the first difference of the logarithm of composite consumer confidence indicator (Normal=100)

The second graph in figure 1 shows the logarithm of GDP and CC. Neither GDP nor CC seems to be stationary, so we proceed using first difference of the logarithm. This transformation appears to be stationary, but the years 2020 and 2021 are rather large outliers and we choose not to include them, to minimize possible normality problems. Therefore, we end up with a sample from 1991 (1) to 2019 (4).

To determine the amount of lags we wish to include in the model, we could use a Box-Jenkins identification method and look at the partial autocorrelation function (PACF). The PACF of Δy_t have the significant lags 1, 2, 3, 6 and 7. However, due to the VAR-model containing simultaneous equations the PACF is not too significant. This could indicate that we should include at least 7 lags when specifying our model.

3 Econometric theory

3.1 The model

We consider a vector autoregressive model of order k given as, VAR(k)-model, since we want to examine the dynamic relationship between two variables. We only look at GDP and CC which means we will use the two dimensional vector $Z_t = (\Delta y_t, \Delta c_t)'$:

$$Z_t = \begin{pmatrix} \Delta y_t \\ \Delta c_t \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \Pi_{11}^1 & \Pi_{12}^1 \\ \Pi_{21}^1 & \Pi_{22}^1 \end{pmatrix} \begin{pmatrix} \Delta y_{t-1} \\ \Delta c_{t-1} \end{pmatrix} + \begin{pmatrix} \Pi_{11}^2 & \Pi_{12}^2 \\ \Pi_{21}^2 & \Pi_{22}^2 \end{pmatrix} \begin{pmatrix} \Delta y_{t-2} \\ \Delta c_{t-2} \end{pmatrix}$$

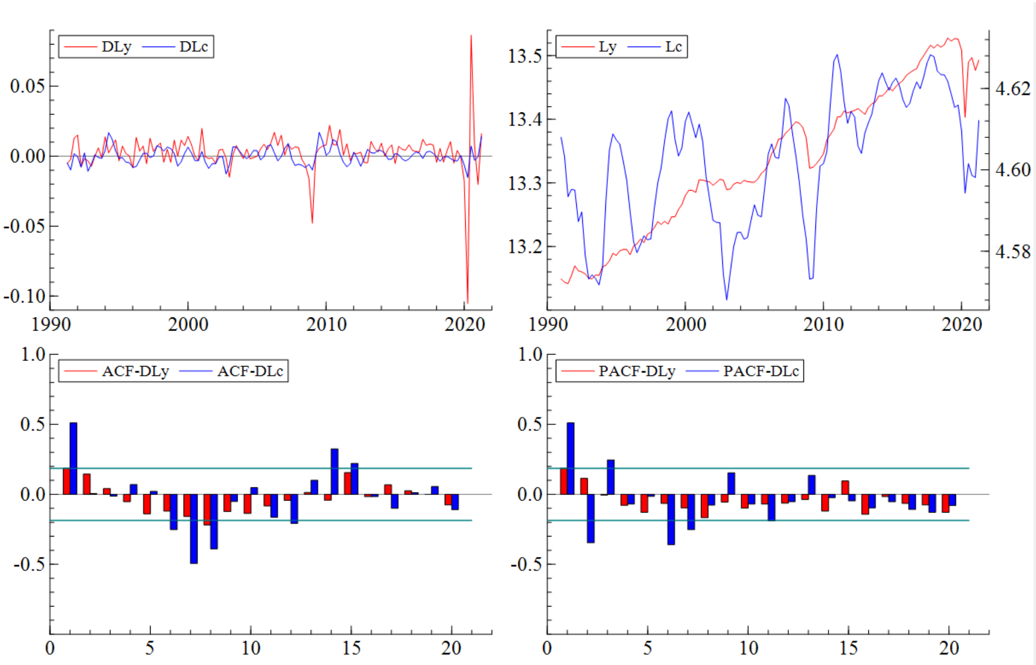


Figure 1: Log GDP/CC, first difference, ACF and PACF

$$+\cdots+\begin{pmatrix}\Pi_{11}^k & \Pi_{12}^k \\ \Pi_{21}^k & \Pi_{22}^k\end{pmatrix}\begin{pmatrix}\Delta y_{t-k} \\ \Delta c_{t-k}\end{pmatrix}+\begin{pmatrix}\Delta\epsilon_{1t} \\ \Delta\epsilon_{2t}\end{pmatrix}$$

The VAR(k)-model is stable if the eigenvalues of the companion matrix are inside the unit circle, which means Z_t is stationary and weakly dependent The companion matrix is given as:

$$\begin{pmatrix}\Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_k \\ I_p & 0 & 0 & \cdots & 0 \\ 0 & I_p & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & I_p & 0\end{pmatrix}$$

We use the Maximum Likelihood Estimator (MLE), which is given by:

$$\hat{\theta}(Z_1, Z_2, ..., Z_T) = \arg\max_\theta L(\theta|Z_1, Z_1, ..., Z_T)$$

OLS will be identical to MLE, when we have Gaussian error terms. We normally assume normality of errors, but this will not be the case when estimating our model. If we want to use MLE, then we must assume normality of errors, so we must instead use the Quasi Maximum Likelihood Estimator (QMLE). The QMLE is not efficient, but it is still consistent and asymptotically normal distributed, as long as the likelihood function is only approximately correct[1].

[1]Nielsen 2021 - Theorem 3.2

There is no directly contemporaneous effect in the model, but if there are any causality between the variables, it will be reflected by the error covariance Ω_{12} in the covariance matrix. But we remember that ϵ_{1t} and ϵ_{2t} can be correlated:

$$E_{t-1}\left(\begin{pmatrix}\epsilon_{1t}\\ \epsilon_{2t}\end{pmatrix}\begin{pmatrix}\epsilon_{1t} & \epsilon_{2t}\end{pmatrix}\right) = \begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix} = \Omega$$

Where $\Omega_{12} = \Omega_{21}$ by symmetry and $E_{t-1}(\cdot) = E(\cdot|I_{t-1})$ denotes the conditional expectation.

3.2 Granger causality

As the VAR-model does not determine the causal direction, we use the Granger causality test to examine if the lagged values of Δc_t in our VAR-model help to predict the second variable Δy_t. Δc_t is said to Granger cause Δy_t if the lagged values of Δc_t are statistically significant in the equation explaining Δy_t. We test the following null-hypothesis of no-Granger causality with a LR-test:

$$\Delta c_t \;\not\Rightarrow\; \Delta y_t : \Pi_{12}^1 = \Pi_{12}^2 = ... = \Pi_{12}^k = 0$$

And we can test that Δy_t does not Granger cause Δc_t by the hypothesis:

$$\Delta y_t \;\not\Rightarrow\; \Delta c_t : \Pi_{21}^1 = \Pi_{21}^2 = ... = \Pi_{21}^k = 0$$

If we reject our first null-hypothesis we say that Δc_t Granger causes Δy_t. If we reject our second null-hypothesis we say that Δy_t Granger causes Δc_t.

3.3 Test for misspecification

We test our model for misspecification by testing for autocorrelation, normality and heteroskedasticity problems, to secure a consistent and unbiased model.

Applied test	Null-hypothesis	Test-statistic
Autocorrelation	There is not autocorrelation at any given lag	$\chi^2(k)$
Heteroskedasticity	There is not heteroskedasticity	$\chi^2(2k)$
Normality	The error terms must be normally distributed	$\chi^2(2)$

Figure 2: Misspecification tests where k is the number of restrictions

When specifying the model, we use the Akaike Information Criterion (AIC) given by:

$$AIC = \log\hat{\sigma}^2 + \frac{2*k}{T}$$

A model is less misspecified the lower the AIC.

3.4 Impulse responses

The moving average representation is given by:

$$Z_t = \epsilon_t + C_1\epsilon_{t-1} + C_2\epsilon_{t-2} + ... + C_{t-1}\epsilon_1 + C_0$$

We have that ϵ is the error term of the reduced form VAR model. It is clear that the impulse responses are just the moving average coefficients. Due to the stationarity condition, the impulse responses will only have a short term effect.

$$\frac{\partial Z_t}{\partial \epsilon_t'} = I_p, \frac{\partial Z_{t+1}}{\partial \epsilon_t'} = C_1, \frac{\partial Z_{t+2}}{\partial \epsilon_t'} = C_2, ...$$

4 Empirical analysis

4.1 Model selection

To determine the number of lags to include, we use a GETS approach starting from 10 lags, and we reduce the number one by one until the last lag is statistical significant. By using the AIC, we end up with a VAR(7) model. We test the model further against lag 1 to 6, and we conclude that we cannot remove any more lags. We end up with the final model with the sample 1993(1) to 2019(4):

$$\begin{pmatrix} \Delta y_t \\ \Delta c_t \end{pmatrix} = \begin{pmatrix} \underset{(0.0038)}{0.034} \\ \underset{(0.0164)}{0.0012} \end{pmatrix} + \begin{pmatrix} \underset{(0.8395)}{0.0216} & \underset{(0.1483)}{0.3072} \\ \underset{(0.7787)}{-0.0131} & \underset{(0.0000)}{0.7572} \end{pmatrix} \begin{pmatrix} \Delta y_{t-1} \\ \Delta c_{t-1} \end{pmatrix} +$$

$$\begin{pmatrix} \underset{(0.6693)}{0.0449} & \underset{(0.2989)}{0.2590} \\ \underset{(0.2498)}{-0.0530} & \underset{(0.0000)}{-0.5882} \end{pmatrix} \begin{pmatrix} \Delta y_{t-2} \\ \Delta c_{t-2} \end{pmatrix} + \begin{pmatrix} \underset{(0.7367)}{0.0352} & \underset{(0.6116)}{-0.1363} \\ \underset{(0.7269)}{-0.0160} & \underset{(0.0126)}{0.2972} \end{pmatrix} \begin{pmatrix} \Delta y_{t-3} \\ \Delta c_{t-3} \end{pmatrix} +$$

$$\begin{pmatrix} \underset{(0.4008)}{-0.0872} & \underset{(0.0499)}{0.5394} \\ \underset{(0.5544)}{0.0267} & \underset{(0.5647)}{-0.0685} \end{pmatrix} \begin{pmatrix} \Delta y_{t-4} \\ \Delta c_{t-4} \end{pmatrix} + \begin{pmatrix} \underset{(0.3807)}{-0.0901} & \underset{(0.2226)}{-0.3277} \\ \underset{(0.3993)}{-0.0378} & \underset{(0.7602)}{0.0356} \end{pmatrix} \begin{pmatrix} \Delta y_{t-5} \\ \Delta c_{t-5} \end{pmatrix} +$$

$$\begin{pmatrix} \underset{(0.8612)}{-0.0179} & \underset{(0.0761)}{0.4254} \\ \underset{(0.1566)}{-0.0635} & \underset{(0.3224)}{-0.1030} \end{pmatrix} \begin{pmatrix} \Delta y_{t-6} \\ \Delta c_{t-6} \end{pmatrix} + \begin{pmatrix} \underset{(0.6482)}{0.0464} & \underset{(0.0593)}{-0.3883} \\ \underset{(0.0133)}{-0.1118} & \underset{(0.0978)}{-0.1485} \end{pmatrix} \begin{pmatrix} \Delta y_{t-7} \\ \Delta c_{t-7} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$$

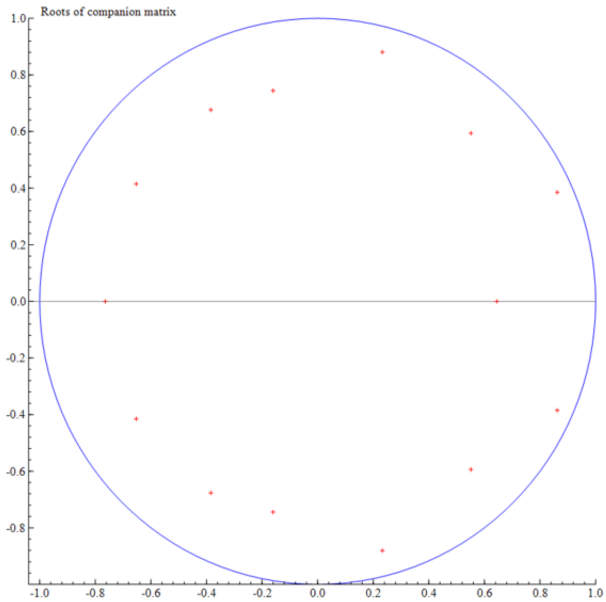
The brackets indicate the P-value. As seen in figure 3, the error terms are not normally distributed, so we end up using the QMLE. The model does not suffer from auto-correlation and heteroskedasticity problems:

To test for stationarity, we calculate the unit roots. We see in figure 4 that the roots of the companion matrix are within the unit circle and the eigenvalues are below 1 in absolute value. We conclude that the model is stationary.

Dy : AR 1-5 test:	F(5,87)	=	2.1312 [0.0691]
Dy : Normality test:	Chi^2(2)	=	40.406 [0.0000]**
Dy : Hetero test:	F(28,78)	=	0.7414 [0.8116]
Dc : AR 1-5 test:	F(5,87)	=	1.0310 [0.4045]
Dc : Normality test:	Chi^2(2)	=	14.969 [0.0006]**
Dc : Hetero test:	F(28,78)	=	1.4955 [0.0845]
Vector AR 1-5 test:	F(20,164)	=	1.5098 [0.0839]
Vector Normality test:	Chi^2(4)	=	52.520 [0.0000]**
Vector Hetero test:	F(84,231)	=	1.0474 [0.3875]

The value in square brackets indicate the P-value with ** indicating 1% confidence interval.

Figure 3: Test results



Eigenvalues of companion matrix:

real	imag	modulus
0.8620	-0.3852	0.9441
0.8620	0.3852	0.9441
0.2330	0.8806	0.9109
0.2330	-0.8806	0.9109
0.5516	0.5940	0.8106
0.5516	-0.5940	0.8106
-0.3842	-0.6766	0.7781
-0.3842	0.6766	0.7781
-0.6527	0.4151	0.7735
-0.6527	-0.4151	0.7735
-0.7642	0.0000	0.7642
-0.1604	-0.7444	0.7615
-0.1604	0.7444	0.7615
0.6448	0.0000	0.6448

Figure 4: Companion matrix with roots and eigenvalues

4.2 Test for Granger Causality

We consider the Granger causality effect between CC and GDP. We use the null-hypothesis described in the econometric theory section:

$$LR(\Delta c_t \not\Rightarrow \Delta y_t) = 20.13^{**}[0.0053] \quad \text{and} \quad LR(\Delta y_t \not\Rightarrow \Delta c_t) = 13.85[0.0538]$$

By this, we conclude that the lags of CC are statistically significant for GDP by using a LR-test with the test statistic $\chi^2(2)$. This means that CC does Granger cause GDP, while GDP does not Granger cause CC.

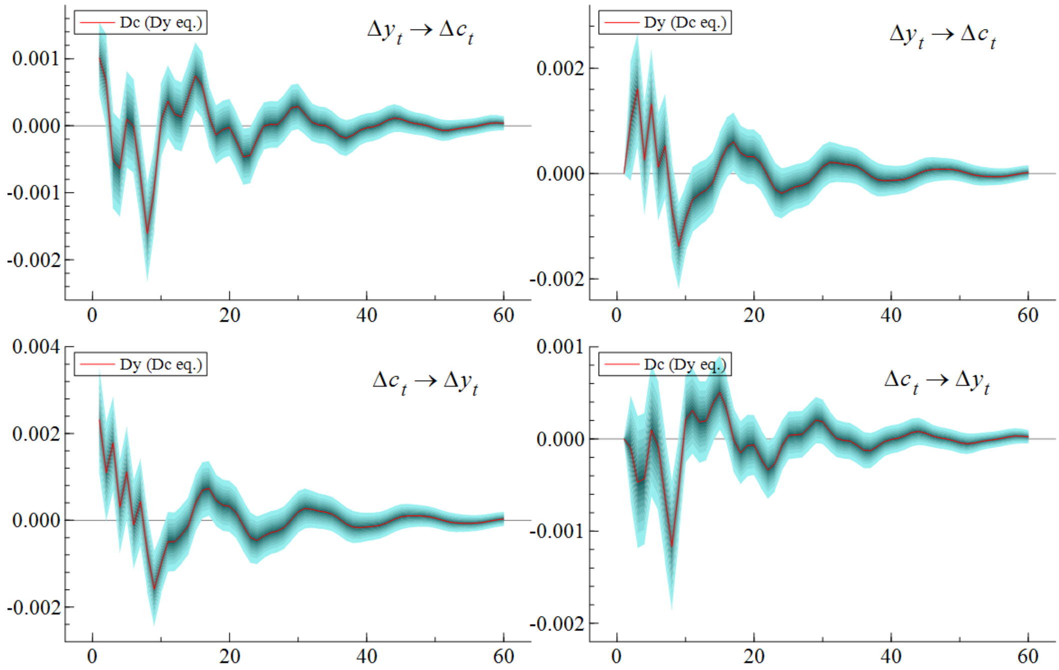


Figure 5: Impulse responses

4.3 Contemporaneous effects and impulse responses

In our estimation we find that $\hat{\rho} = corr(\epsilon_{1t}\epsilon_{2t}) = 0.2984$. We calculate the critical value for the estimated correlation to be:

$$\frac{1.96}{\sqrt{T}} = \frac{1.96}{\sqrt{108}} = 0.188$$

The estimated correlation is therefore significantly different from zero, and thus we have a contemporaneous effect.

Since we have a contemporaneous effect, interpreting a shock to ϵ_{1t} is problematic, since ϵ_{1t} and ϵ_{2t} correlate. We instead show the orthogonal impulse response functions of the reduced VAR model in figure 5.

In the first two graphs of figure 5, we impose the restriction that $\Delta y_t \rightsquigarrow \Delta c_t$. This means that Δy_t affects Δc_t in period t. In the last two graph we have the opposite effect, such that $\Delta c_t \rightsquigarrow \Delta y_t$. It is clear to see that the contemporaneous effect from Δc_t to Δy_t is larger compared to the opposite. This indicates that CC have larger contemporaneous effect on GDP than the other way.

5 Discussion and conclusion

The direction of the Granger causality could be explained by households consuming/spending according to their confidence in the economy, so that they will spend more when CC is high and less when CC is low. We do not discover a causal direction the other way.

As the Granger causality only includes preceding periods in the explanatory variables, we also study the contemporaneous effect to discover the impact in period t. We determine that there are contemporaneous effects. The causal directions therefore cannot be determined in the period t. It is not possible to assume $\Delta y_t \not\Rightarrow \Delta c_t$ in period t. However, we find that there are larger contemporaneous effects from CC on GDP compared to the other way around.

Contrary to Utaka (2003) we find that consumer confidence not only have a short-run impact on GDP, but also has a long-run impact on GDP. However, Utaka only includes 2 lags in his model and uses data for Japan compared to Germany, while ours includes 7, so it is possible we have overfitted our model.