Assignment #3: Empirical Relevance of the Purchasing Power Parity Relationship

1 Introduction

In this paper we test the purchasing power parity (PPP) theory by analyzing the consumer price index (CPI) in Mozambique and South Africa. We also include the bilateral exchange rate between the 2 countries. We choose a Vector Autoregressive Model (VAR(3)) model and use it to write up the Vector Error Correction Model (VECM) to test for Co-integration. Using a trace test we find that the VECM with rank 1 is significant. Furthermore, we show that there is a long run relationship between the 2 price indexes. We use impulse response functions to investigate the short run Co-integration effects. The VECM finds a weak version of PPP to hold, where the bilateral exchange rate has an effect in the short run.

2 Description of data

In this paper we use data for the CPI in Mozambique (CPIMZM) and South Africa (CPIZAR), as well as the bilateral exchange-rate (MZM_ZAR) which is denominated as meticais per rand. All the variables are taken from the Mozambique macro database. The variables are recorded monthly from 2004:1 to 2021:9, which will be our sample.
We define the variables:

$$p_t = \log(\text{CPIMZM}_t)$$
$$p_t^* = \log(\text{CPIZAR}_t)$$
$$e_t = \log(\text{MZM\_ZAR}_t)$$

We can see from the graphical analysis in figure 1 that our 3 variables p_t, p_t^* and e_t are not stationary, as all three seems to have an upward trend in the period, and thus not fluctuating around a constant mean. Furthermore, the two price indexes show the same growth in the period, which could indicate co-integration of the two time-series. We have also included the seasonal subplots, which indicate seasonality. Therefore, we use seasonal dummies to adjust for that.

3 Econometric Theory

3.1 PPP

According to the strict version of PPP, we would expect the following relation to hold in the long run, where U_t is a constant and $u_t = \log(U_t)$:

$$U_t = \frac{P_t}{P_t^* E_t} \Leftrightarrow u_t = p_t - p_t^* - e_t$$

This shows that the exchange rate between two currencies should allow the same amount of goods to be purchased. The weak version of PPP would instead suggest the following relationship:

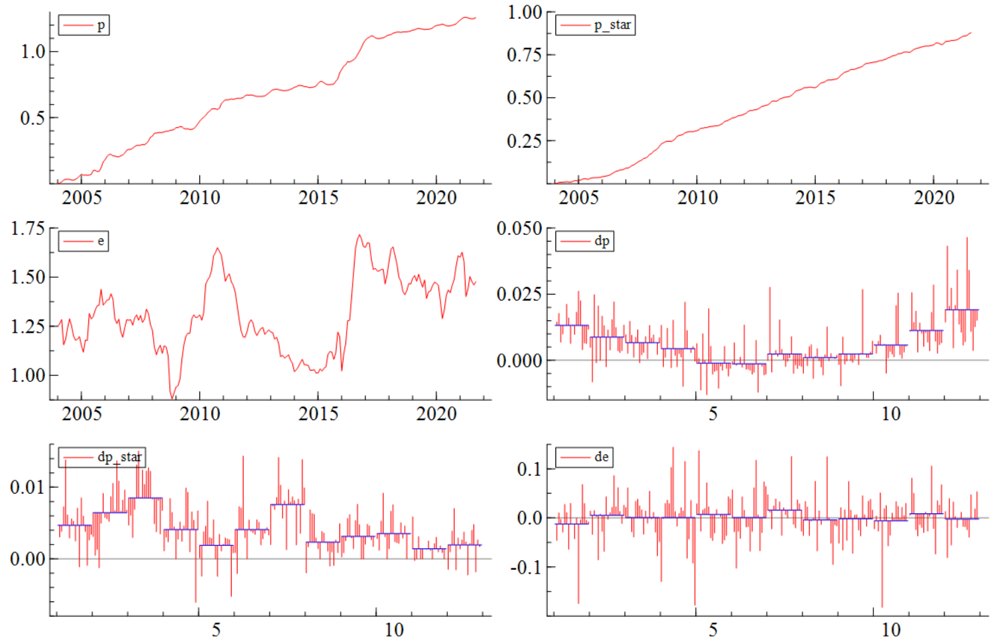

Figure 1: Logarithmic transformations of the time series and seasonal subplots

$$p_t^* = \beta_1 p_t - \beta_2 e_t + u_t$$

3.2 The Model

We consider the trivariate vector of variables:

$$X_t = \begin{pmatrix} p_t \\ p_t^* \\ e_t \end{pmatrix}$$

We consider the VAR-model, because it describes the dynamic relationship of the three variables. We choose a VAR(3) model for a 3-dimensional vector:

$$X_t = \Pi_{t-1} X_{t-1} + \Pi_2 X_{t-2} + \Pi_3 X_{t-3} + \mu + \epsilon_t, \;\; t = 1, 2, ..., T,$$

We define the following constant and error terms $\mu = (\mu_1, \mu_2, \mu_3)'$ and $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3)'$. For the likelihood analysis, we assume:

$$\epsilon_t | X_{t-1}, X_{t-2}, X_{t-3} \overset{d}{=} N(0, \sigma)$$

We write up the VECM form to test for Co-integration.

$$X_t - X_{t-1} = (\Pi_1 + \Pi_2 + \Pi_3 - I_p) X_{t-1} - (\Pi_2 + \Pi_3)(X_{t-2} - X_{t-1}) - \Pi_3 (X_{t-3} - X_{t-2}) + \mu + \epsilon_t$$

$$\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \mu + \epsilon_t$$

where $\Gamma_1 = -(\Pi_2 + \Pi_3)$, $\Gamma_2 = -\Pi_3$ and where the VECM has unit roots if $|\Theta(1)| = |I_p - \Pi_1 - \Pi_2 - \Pi_3| = 0$.

3.3 Estimator and misspecification-test

We consider using the Maximum Likelihood Estimator (MLE) to estimate our VAR-model. However, in the case of a non-normal distribution of errors we would use the Quasi-MLE(QMLE) instead. We test for autocorrelation, misspecification and normality issues by applying the tests denoted in assignment 2. If the normality is rejected, then QMLE will be used as estimator. The QMLE won't be efficient, but is consistent if standard assumptions hold. To determine the best fit for our model we apply the Akaike Information Criterion (AIC).

3.4 Stationarity, Co-integration and Testing

To check for Co-integration between our variables, we need to determine the rank of Π, so we decompose Π into $\alpha\beta'$. If X_t is a full rank Matrix, α and β' are both 3×3 matrices. If X_t is unit root and there is no co-integration, then there are no stationarity between the variables of the model. If we have rank 1, then the α_1 and β_1 matrices are 3×1, while α_2 and β_2 are 3×2 when we have rank 2. We have three nested models when testing for Unit-Roots and Co-integration:

$$\begin{aligned}
H_0 : \Delta X_t &= \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \mu + \epsilon_t \\
H_1 : \Delta X_t &= \alpha_1 \beta_1' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \mu + \epsilon_t \\
H_2 : \Delta X_t &= \alpha_2 \beta_2' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \mu + \epsilon_t \\
H_3 : \Delta X_t &= \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \mu + \epsilon_t
\end{aligned}$$

We can test for the Co-integration rank using likelihood ratio (LR) tests:

$$LR(H_0|H_2) = -2(\log L(H_0) - \log L(H_2))$$

$$LR(H_1|H_2) = -2(\log L(H_1) - \log L(H_2))$$
$$LR(H_2|H_3) = -2(\log L(H_2) - \log L(H_3))$$

where $\log(H_i)$ denotes the maximized log-likelihood value, $i = 0, 1, 2, 3$.

These test are known as the trace test statistics and they follow a Dickey-Fuller distribution when $T \to \infty$. Furthermore we also test for general misspecification such as autocorrelation, normality and heteroskedasticity in our VAR-model.

To examine the Co-integration in the short run we use the moving average representation is given by:

$$X_t = \epsilon_t + C_1 \epsilon_{t-1} + C_2 \epsilon_{t-2} + ... + C_{t-1} \epsilon_1 + C_0$$

Impulse responses are given by:

$$\frac{\partial X_t}{\partial \epsilon_t'} = I_p, \frac{\partial X_{t+1}}{\partial \epsilon_t'} = C_1, \frac{\partial X_{t+2}}{\partial \epsilon_t'} = C_2, ...$$

4 Empirical analysis

4.1 Model Selection

When specifying the model, we start by testing a VAR-model with 13 lags against a VAR-model with fewer lags. By using the AIC, we find that the first three lags are significant, so we end up with a VAR(3) model with centered seasonal dummies and using the full sample from 2004:4 and not 2004:1 due to the lags. By doing mispecification testing, we find that the model suffers from normality problems, which will be addressed later on. It also suffers from heteroskedasticity, which is corrected for by using bootstrap rank testing.

We test the null hypothesis of reduced rank in figure 2:

Rank	Trace	5% critical value	Crit*5%	P-value	P-value*
0	55.42	29.80	42.62	[0.000]**	[0.003]**
1	9.04	15.41	23.07	[0.368]	[0.499]
2	0.84	3.84	8.12	[0.361]	[0.441]

Figure 2: Likelihood test for co-integration rank from 2005 (2) without dummies

We now test rank 0 against rank 3. The LR-test value is $LR(H_0|H_3) = 55.42$, and the critical valued based on bootstrap is 42.62. We can reject the null hypothesis of rank 0 against rank 3. We instead test rank 1 against rank 3, and we cannot reject this null hypothesis. This suggest that we should use a VECM with rank 1. The Co-integration vectors of this model is shown in graph 4-6 in figure 4, and the β_2 and β_3 Co-integration vectors appear to be random walks, while β_1 seems stationary. Our test results align with the graphical analysis.

The problem with normality could be addressed by introducing residual dummies. Using the above approach again, we find the trace results in figure 3.

Rank	Trace	5% critical value	Crit*5%	P-value	P-value*
0	80.69	29.80	42.92	[0.000]**	[0.000]**
1	25.89	15.41	24.13	[0.001]**	[0.005]**
2	07.40	3.84	7.90	[0.007]**	[0.015]*

Figure 3: Likelihood test for co-integration rank from 2005 (2) with dummies

This implies that our model is of full rank according to the trace statistic. However, comparing with graph 2 and 3 in figure 4, the β_2 and β_3 vectors likely are not stationary. This contradiction between our trace tests and our graphical analysis could be caused by a random walk becoming stationary due to the use of too many residual dummies. We therefore chose not to include any residual dummies but we keep the seasonal dummies in our further analysis.

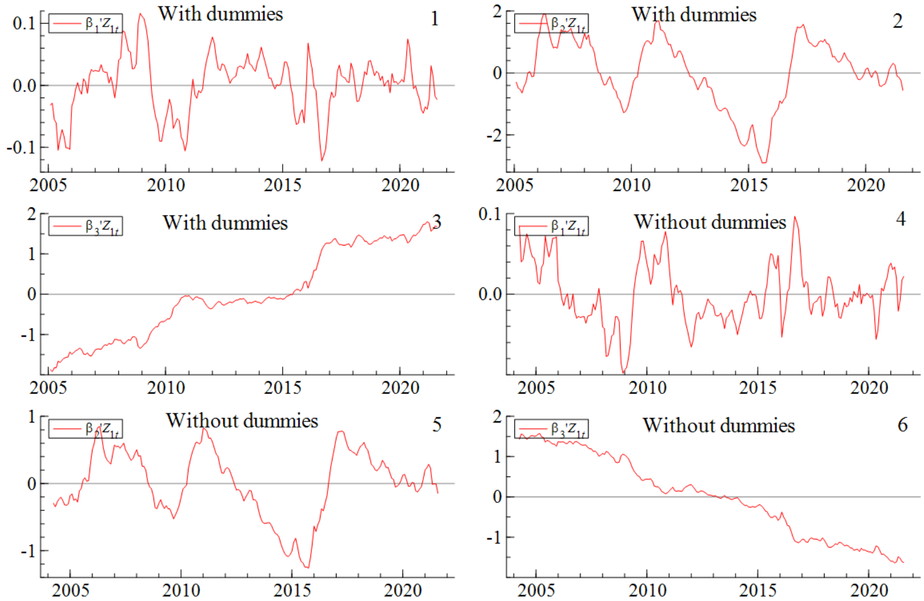

Figure 4: Co-integrated vectors

4.2 Short- and long-run effects

We restrict the VECM to only include rank 1 and get the following α- and β-coefficients with T-values in parenthesis:

$$\alpha\beta'\Delta X_t = \begin{pmatrix} \underset{(3.9)}{0.0562} \\ \underset{(-5.3)}{-0.0349} \\ \underset{(-1.8)}{-0.206} \end{pmatrix} \begin{pmatrix} \underset{(-65.3)}{-0.795} & 1 & \underset{(10.8)}{0.274} \end{pmatrix} \begin{pmatrix} \Delta p_t \\ \Delta p_t^* \\ \Delta e_t \end{pmatrix}$$

$$= \begin{pmatrix} \underset{(-3.9)}{-0.0447} & \underset{(3.9)}{0.0562} & \underset{(3.9)}{0.0154} \\ \underset{(5.3)}{0.0278} & \underset{(-5.3)}{-0.0349} & \underset{(-5.3)}{-0.0096} \\ \underset{(1.8)}{0.164} & \underset{(-1.8)}{-0.206} & \underset{(-1.8)}{-0.0565} \end{pmatrix} \begin{pmatrix} \Delta p_t \\ \Delta p_t^* \\ \Delta e_t \end{pmatrix}$$

It appears that e_t does not error correct, since it has a T-value of -1.8. We impose the restriction that $\alpha_3 = 0$ which gives us a test restriction of $\chi^2(1) = 2.6082$ [0.1063] with p-values reported in square brackets. We cannot reject our null hypothesis that $\alpha_3 = 0$, which means e_t does not seem to error correct. Instead, it appears that both p_t and p_t^* error corrects since we cannot restrict their α-coefficients to be zero. This suggest that the two variables are Co-integrating in the long run. We also note that the α-coefficients of p_t is larger than the coefficient of p_t^*. This suggest that the price index in Mozambique error corrects more in the short run to the price index in South Africa than the other way around. In general we have that p_t and p_t^* share the same underlying stochastic trend.

The long run normalized vector when $\alpha_3 = 0$ is given by:

$$p_t^* = \beta_1 p_t - \beta_2 e_t + u_t \Leftrightarrow p^* = 0.79 * p_t + 0.254 e_t + u_t$$

We test $\beta_1 = 1$ and $\beta_2 = 0$, which is the strict version of PPP. We reject this using a $\chi^2(1)$ LR-test with a test value of 30.519, which rejects that the strict version of PPP. Therefore, since we have established long run co-integration, a weak version of the PPP will hold. A 1% increase in South Africa's CPI (p^*) results in an increase of 0.79% in Mozambique's CPI (p). Likewise a 1% increase in CPIZAR will have an increase of 0.254% in the bilateral exchange rate between Mozambique's and South Africa's currencies.

To visualize the results, we also find the following impulse response functions in figure 5.

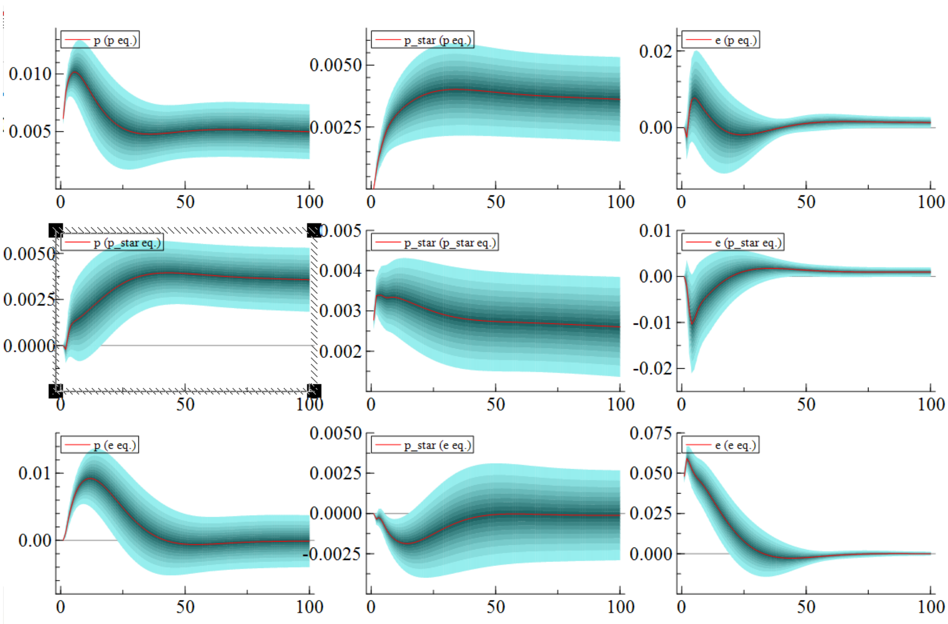

Figure 5: Impulse response functions based on standard errors

In the short run, p_t^* has a clear effect on p_t, while the opposite also appears to be true. The same is true when we look at the long run effect. A shock to e_t has a small short run effect on p_t^* and p_t, but the long run effect does not seem to be significant. A shock to p_t^* and p_t^* affects both currencies in the short and the long run, which corresponds to our Co-integration analysis. The time before the model converges appears to be approximately 50 months.

5 Discussion and concluding remarks

As seen in the empirical analysis, dummies should be approached cautiously, as including an excessive amount of dummies could make a random walk process stationary.

It is a trade-off between fixing normality problems with dummies and risking imposing stationarity on non-stationary time series. Our model suffers from normality errors, which could result in our maximum likelihood estimator not being consistent or efficient, which would require our estimator to be QMLE instead.

Instead of using VECM for Co-integration, we could also have used the Engle-Granger approach or Co-integration with the Autoregressive Distributed Lag (ADL) model. However, if the variables do not co-integrate, we could end up with spurious correlation when using both the Engle-Granger approach and the ADL model. Co-integration with ADL would have been risky due to the fact that we have 3 variables to estimate and Co-integrate, which is not suitable for this approach, as it assumes only one of the variables to error correct and only one Co-integration relationship. However, it turns out that we only have one variable error correcting and only one Co-integration, which would allow us to use this approach, but this is not known a priori.

Corbae and Ouliaris (1988) rejects the null hypothesis of the PPP to hold in the long run, while Kim (1990) find the theory to hold. Taylor (1988) find the theory to be very unlikely to hold. This both aligns and contrasts with our results. It is difficult to compare results since our data samples are quite different.

Our study finds that the price-indexes of the two nations error-corrects and thus Co-integrate. We determine the Co-integration to be of rank one, which result in the PPP-theory to hold weakly in the long run. We also find that the price-indexes of the two nations affect each other in the short run, and we find that the bilateral exchange rate between the two currencies only slightly affect the price-indexes in the short run, which it does not do in the long run.