

# Macroeconomics III

## Lecture 11

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## Finding the expected loss

Need to prove:

$$\begin{aligned} E[L] &= \\ &= \frac{1}{2} \left[ \begin{aligned} &\psi^2 + \psi_\theta^2 \sigma_\theta^2 + (1 + \psi_\nu)^2 \sigma_\nu^2 + \psi_\epsilon^2 \sigma_\epsilon^2 + \sigma_\mu^2 + \bar{\pi}^2 - 2\psi\bar{\pi} \\ &+ \lambda(\sigma_\theta^2 + (1 + \psi_\nu)^2 \sigma_\nu^2 + (\psi_\epsilon - 1)^2 \sigma_\epsilon^2 + \sigma_\mu^2 + \bar{x}^2) \end{aligned} \right] \end{aligned}$$

## Finding the expected loss- Step 1

- Let's start by collecting stochastic variables and *scalars* within the squared terms of the expected loss

$$E[L] = \frac{1}{2} E \left[ \begin{array}{l} (\psi - \bar{\pi} + \psi_{\theta}\theta + (1 + \psi_{\nu})\nu + \psi_{\epsilon}\epsilon + \mu)^2 \\ + \lambda(-\bar{x} + \theta + (\psi_{\nu} + 1)\nu + (\psi_{\epsilon} - 1)\epsilon + \mu)^2 \end{array} \right]$$

- Thus:

$$= \frac{1}{2} E \left[ \begin{array}{l} (\psi - \bar{\pi})^2 + (\psi_{\theta}\theta + (1 + \psi_{\nu})\nu + \psi_{\epsilon}\epsilon + \mu)^2 \\ + 2(\psi - \bar{\pi})(\psi_{\theta}\theta + (1 + \psi_{\nu})\nu + \psi_{\epsilon}\epsilon + \mu) \\ + \lambda\bar{x}^2 + \lambda(\theta + (\psi_{\nu} + 1)\nu + (\psi_{\epsilon} - 1)\epsilon + \mu)^2 \\ - 2\lambda\bar{x}(\theta + (\psi_{\nu} + 1)\nu + (\psi_{\epsilon} - 1)\epsilon + \mu) \end{array} \right]$$

## Finding the expected loss - Step 2

Take expectations:

- *Line 1:*
  - $E[(\psi - \bar{\pi})^2] = (\psi - \bar{\pi})^2$
  - $E[(\psi_\theta \theta + (1 + \psi_\nu) \nu + \psi_\epsilon \epsilon + \mu)^2] = \psi_\theta^2 \sigma_\theta^2 + (1 + \psi_\nu)^2 \sigma_\nu^2 + \psi_\epsilon^2 \sigma_\epsilon^2 + \sigma_\mu^2$
- *Line 2:*  $E[2(\psi - \bar{\pi})(\psi_\theta \theta + (1 + \psi_\nu) \nu + \psi_\epsilon \epsilon + \mu)] = 0$
- *Line 3:*
  - $E[\lambda \bar{x}^2] = \lambda \bar{x}^2$
  - $E[\lambda(\theta + (\psi_\nu + 1) \nu + (\psi_\epsilon - 1) \epsilon + \mu)^2] =$   
 $\lambda [\sigma_\theta^2 + (1 + \psi_\nu)^2 \sigma_\nu^2 + (\psi_\epsilon - 1)^2 \sigma_\epsilon^2 + \sigma_\mu^2]$
- *Line 4:*  $E[-2\lambda \bar{x}(\theta + (\psi_\nu + 1) \nu + (\psi_\epsilon - 1) \epsilon + \mu)] = 0$

## Finding the expected loss - Step 3

Collect terms:

$$E[L] = \frac{1}{2} \left[ \begin{array}{l} \psi^2 + \bar{\pi}^2 - 2\psi\bar{\pi} + \psi_{\theta}^2\sigma_{\theta}^2 + (1 + \psi_{\nu})^2\sigma_{\nu}^2 + \psi_{\epsilon}^2\sigma_{\epsilon}^2 + \sigma_{\mu}^2 \\ + \lambda(\bar{x}^2 + \sigma_{\theta}^2 + (1 + \psi_{\nu})^2\sigma_{\nu}^2 + (\psi_{\epsilon} - 1)^2\sigma_{\epsilon}^2 + \sigma_{\mu}^2) \end{array} \right]$$

# January 2017 Exercise 2

## Question 1

*Show that the optimal policy under commitment implies  $\pi_t^C = 0$  and  $x_t^C = \theta_t$  [hint: i) recall that the loss function is quadratic, thus the optimal policy rule is linear and can be guessed to be of the form  $\pi_t = \psi + \psi_\theta \theta_t$ ; ii) recall that the loss to be minimized is the unconditional one].*

# January 2017 Exercise 2

## Question 1

- Given the linear rule  $\pi_t = \psi + \psi_\theta \theta_t$ , as well as the fact that  $\theta_t$  is observed by both the public and the policy maker before expectations are formed, output is determined as follows:

$$x_t = \theta_t + \pi_t - \pi_t^e = \theta_t + \psi + \psi_\theta \theta_t - (\psi + \psi_\theta \theta_t) = \theta_t$$

- Thus, the expected loss reads as:

$$\begin{aligned} E[L(\pi_t, x_t)] &= \frac{1}{2} E \left[ \left( \underbrace{\psi + \psi_\theta \theta_t}_{=\pi_t} \right)^2 + \lambda \left( \underbrace{\theta_t}_{=x_t} - \bar{x} \right)^2 \right] \\ &= \frac{1}{2} E \left[ \psi^2 + 2\psi\psi_\theta \theta_t + \psi_\theta^2 \theta_t^2 + \lambda (\theta_t^2 - 2\bar{x}\theta_t + \bar{x}^2) \right] \\ &= \frac{1}{2} \left[ \begin{array}{l} \psi^2 + 2\psi\psi_\theta E[\theta_t] + \psi_\theta^2 E[\theta_t^2] \\ + \lambda (E[\theta_t^2] - 2\bar{x}E[\theta_t] + \bar{x}^2) \end{array} \right] \end{aligned}$$

# January 2017 Exercise 2

## Question 1

- Taking the first order conditions of  $E[L(\pi_t, x_t)]$  with respect to  $\psi$  and  $\psi_\theta$  we obtain:

$$\frac{\partial E[L(\pi_t, x_t)]}{\partial \psi} = 0 : \psi + \psi_\theta E[\theta_t] = 0$$

$$\frac{\partial E[L(\pi_t, x_t)]}{\partial \psi_\theta} = 0 : \psi E[\theta_t] + \psi_\theta E[\theta_t^2] = 0$$

- Thus, the expected loss is minimized by setting  $\psi = \psi_\theta = 0$ , which implies  $\pi_t^C = 0$  and  $x_t^C = \theta_t$ .



# January 2017 Exercise 2

## Question 2

*Show that the optimal policy under discretion implies  $\pi_t^D = -\lambda (\theta_t - \bar{x})$  and  $x_t^D = \theta_t$ . The inflation bias increases in the target  $\bar{x}$ : explain why.*

# January 2017 Exercise 2

## Question 2

- When the central bank conducts a discretionary policy, the inflation rate is chosen after expectations are formed. Hence, the goal of the central bank is to minimize the loss function, i.e. the monetary policy should be ex post optimal, given  $\pi_t^e$ . Under this assumption, the problem reads as

$$\min_{\pi_t} \frac{1}{2} \left[ \pi_t^2 + \lambda (\theta_t + \pi_t - \pi_t^e - \bar{x})^2 \right]$$

- The first order condition for this problem reads as:

$$\frac{\partial L(\pi_t, x_t)}{\partial \pi_t} = 0 : \pi_t + \lambda (\theta_t + \pi_t - \pi_t^e - \bar{x}) = 0 \Leftrightarrow \pi_t^D = \frac{\lambda}{1 + \lambda} (\pi_t^e - \theta_t + \bar{x})$$

- Thus, the expected rate of inflation is found by taking expectations:

$$E \left[ \pi_t^D \mid \theta_t \right] = \frac{\lambda}{1 + \lambda} E_t \left[ \pi_t^e - \theta_t + \bar{x} \right] = \frac{\lambda}{1 + \lambda} \left( E_t \left[ \pi_t^D \mid \theta_t \right] - \theta_t + \bar{x} \right)$$

which implies  $E \left[ \pi_t^D \mid \theta_t \right] = -\lambda (\theta_t - \bar{x})$

# January 2017 Exercise 2

## Question 2

- Therefore:

$$\pi_t^D = \frac{\lambda}{1 + \lambda} \left( \underbrace{-\lambda (\theta_t - \bar{x})}_{=\pi_t^e} - \theta_t + \bar{x} \right) = -\lambda (\theta_t - \bar{x})$$
$$x_t^D = \theta_t$$

- The excessively high equilibrium inflation associated with the inflation bias problem results from the combination of a lack of commitment and central bank's temptation to temporarily boost the economy beyond its potential level. The latter incentive is embodied by the condition  $\bar{x} > \theta$ . This makes it clear why raising  $\bar{x}$  increases the temptation of the central bank to generate excess inflation in the vain attempt to stimulate real activity.

# January 2017 Exercise 2

## Question 3

*Assume that potential output cannot be observed before expectations are formed. The goal of the central bank is still to minimize the loss function. However, the monetary policy stance should now result as ex-post optimal given both  $\pi_t^e$  and  $\theta_t$  (as the latter is not observed until after expectations are formed). Assume that  $E[\theta_t] = 0$ . Show that the optimal policy under discretion now implies  $\pi_t^{D*} = \frac{\lambda}{1+\lambda} [\bar{x}(1+\lambda) - \theta_t]$  and  $x_t^{D*} = \frac{1}{1+\lambda} \theta_t$ . Under  $\lambda = 0$  it is possible to ensure that  $\pi_t^{D*} = \pi_t^C$  and  $x_t^{D*} = x_t^C$ . Explain why this is the case.*

# January 2017 Exercise 2

## Question 3

- Once again, when the central bank conducts a discretionary policy, the inflation rate is chosen after expectations are formed. Hence, the goal of the central bank is to minimize the loss function, i.e. the monetary policy should be ex post optimal, now given  $\pi_t^e$  and  $\theta_t$ , as the latter is not observed. Under this assumption, the problem reads as

$$\min_{\pi_t} \frac{1}{2} \left[ \pi_t^2 + \lambda (\theta_t + \pi_t - \pi_t^e - \bar{x})^2 \right]$$

- The first order condition for this problem reads as:

$$\begin{aligned} \frac{\partial L(\pi_t, x_t)}{\partial \pi_t} &= 0 : \pi_t + \lambda (\theta_t + \pi_t - \pi_t^e - \bar{x}) = 0 \\ \Leftrightarrow \pi_t^D &= \frac{\lambda}{1 + \lambda} (\pi_t^e + \bar{x} - \theta_t) \end{aligned}$$

# January 2017 Exercise 2

## Question 3

- Now, the expected rate of inflation is found by taking unconditional expectations (as  $\theta_t$  is not observed before expectations are formed):

$$E[\pi_t^D] = \frac{\lambda}{1+\lambda} E[\pi_t^e - \theta_t + \bar{x}] = \frac{\lambda}{1+\lambda} (E[\pi_t^D] + \bar{x})$$

which implies  $E[\pi_t^D] = \lambda \bar{x}$

- Therefore:

$$\pi_t^{D*} = \frac{\lambda}{1+\lambda} \left( \underbrace{\lambda \bar{x}}_{=\pi_t^e} + \bar{x} - \theta_t \right) = \frac{\lambda}{1+\lambda} [\bar{x}(1+\lambda) - \theta_t]$$

$$x_t^{D*} = \theta_t + \frac{\lambda}{1+\lambda} [\bar{x}(1+\lambda) - \theta_t] - \lambda \bar{x} = \frac{1}{1+\lambda} \theta_t$$

# January 2017 Exercise 2

## Question 3

- As we set  $\lambda = 0$ , the policy maker does not face a real activity stabilization objective, so that there is no temptation to inflate the economy to raise output above the target
- Thus, no matter the information structure, output will always be equal to  $\theta_t$ , and thus to its solution under commitment
- The same holds true for the rate of inflation

# Open (real) economy: a primer

- Topics:
  - Consumption and capital accumulation in a small open economy (SOE)
  - Current account dynamics and net foreign assets
  - Real exchange rate (RER) determination and dynamics in a two-sector open economy
  - Source: DN 7.1-7.2



## Definitions

- In an open economy investment is no longer constrained by saving, but the economy must satisfy an intertemporal budget constraint
- Let  $tb_t \equiv y_t - c_t - i_t$  denote the *trade balance* (the net amount of output the economy transfers to foreigners)
- As for the *net foreign asset position*,  $nfa_t$ :

$$nfa_t \equiv a_t - k_t$$

- Current account (absent capital gains or losses) in a small open economy:

$$ca_t \equiv tb_t + r_t nfa_t = \Delta nfa_{t+1}$$

- A positive trade balance implies an equal-sized increase in net foreign assets, as it forces foreigners to borrow from domestic agents

# Firms

- Usual assumptions (see Lecture 2)
- The representative firm maximizes profits:

$$\max_{K_t, L_t} f(K_t, L_t) - r_t K_t - w_t L_t$$

- FOCs:

$$f_K(k_t, 1) = r_t$$

$$f_L(k_t, 1) = w_t$$

which define demand the functions for capital and labor

- Note that with constant world rental rates and time invariant technology the domestic capital stock and the wage are constant at all times

# Households

Using the dynamic budget constraint

- The representative household solves

$$\begin{aligned} \max_{c_t, k_{t+1}} \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t.} \quad & a_{t+1} = a_t R_t + w_t - c_t \end{aligned}$$

- Thus:

$$\begin{aligned} a_{t+1} &= a_t R_t + w_t - c_t \\ u'(c_t) &= \beta R_{t+1} u'(c_{t+1}) \\ \text{where } R_t &= 1 + r_t - \delta \end{aligned}$$

# Current account dynamics

The current account satisfies:

$$\begin{aligned}tb_t + (R_t - 1) nfa_t &= f(k_t, 1) - \underbrace{c_t}_{a_t R_t + w_t - a_{t+1}} - [k_{t+1} - (1 - \delta)k_t] + (R_t - 1) \underbrace{nfa_t}_{a_t - k_t} \\&= f(k_t, 1) - a_t R_t - w_t + \underbrace{a_{t+1} - k_{t+1}}_{= nfa_{t+1}} + (1 - \delta)k_t + R_t a_t \underbrace{- a_t + k_t}_{= -nfa_t} - R_t k_t \\&= f(k_t, 1) - w_t + \underbrace{(1 - R_t - \delta)k_t}_{= -r_t} + nfa_{t+1} - nfa_t = \Delta nfa_{t+1}\end{aligned}$$

# Consumption and capital accumulation in a small open economy

- In an open economy consumption and investment decisions are not *interdependent*, as in the closed economy
- Instead, they are *recursive*:
  - Investment is chosen first, such that in each period the marginal product of capital (after depreciation) equals the world interest rate  $R \Rightarrow$  The  $k_{t+1}$  sequence determines the wealth of the household
  - Given wealth and the world interest rate, consumption follows from the intertemporal budget constraint and the Euler equation

## Equilibrium consumption

- To solve for equilibrium consumption, we iterate the dynamic budget constraint forward (assume  $R_t = R = \beta^{-1}$ ):

$$c_t = \rho R a_t + \rho \sum_{s=0}^{\infty} R^{-s} w_{t+s}$$

where  $\rho \equiv \left( \sum_{s=0}^{\infty} R^{-s} \right)^{-1} = (R - 1) / R$

- As usual, equilibrium consumption reflects lifetime wealth
- Unlike in closed-economy general equilibrium models, however, the interest rate is exogenous and the open capital account allows for domestic saving and investment and, thus, consumption and capital accumulation, to be decoupled

# Consumption dynamics and RER determination

## Assumptions

- Suppose now an endowment-SOE with two components:
  - A non-tradable component,  $w_t^N$
  - A tradable component,  $w_t^T$
- Non-tradables only can be consumed domestically (think of them as services), while tradables can both be consumed domestically and shipped abroad at no cost
- The tradable good serves as numeraire and the price of non-tradables is denoted  $p_t$

# Consumption dynamics and RER determination

## Model

- Household consumption is a CES aggregator of the tradable and the non-tradable good:

$$c_t = c \left( c_t^N, c_t^T \right)$$

- The price of  $c_t$ , which increases in  $p_t$ , is denoted by  $\mathcal{P}_t$
- The RER is the price of one unit of domestic consumption relative to the price of a consumption unit abroad, which we normalize to unity
- The RER thus equals  $\mathcal{P}_t$  and increases in  $p_t$ ; thus, a rising price of non-tradables implies a RER appreciation
- Households maximize the discounted stream of utility, such that

$$a_{t+1} = a_t R_t + w_t^T + w_t^N p_t - c_t^T - p_t c_t^N$$



# Consumption dynamics and RER determination

## Equilibrium

- From the FOCs (prove it):

$$\begin{aligned}u'(c_t) \mathcal{P}_t &= \beta R_{t+1} u'(c_{t+1}) \mathcal{P}_{t+1} \\u'(c_t) c_T(c_t^N, c_t^T) &= \beta R_{t+1} u'(c_{t+1}) c_T(c_{t+1}^N, c_{t+1}^T) \\p_t &= \frac{c_N(c_t^N, c_t^T)}{c_T(c_t^N, c_t^T)}\end{aligned}$$

where  $c_T = \partial c(c_t^N, c_t^T) / \partial c_t^T$  and  $c_N = \partial c(c_t^N, c_t^T) / \partial c_t^N$

# Consumption dynamics and RER determination

## Equilibrium

- Assume  $R_t = \beta^{-1}$ ,  $w_t^N = w^N$  and domestic market clearing ( $c_t^N = w^N$ ):

$$\begin{aligned}u'(c_t) \mathcal{P}_t &= u'(c_{t+1}) \mathcal{P}_{t+1} \\u'(c_t) c_T(w^N, c_t^T) &= u'(c_{t+1}) c_T(w^N, c_{t+1}^T) \\p_t &= \frac{c_N(w^N, c_t^T)}{c_T(w^N, c_t^T)}\end{aligned}$$

- The first two conditions imply

$$\frac{c_T(w^N, c_{t+1}^T)}{c_T(w^N, c_t^T)} = \frac{\mathcal{P}_{t+1}}{\mathcal{P}_t}$$

- What happens if  $c_{t+1}^T/c_t^T$  increases? Effect on the *rate of acceleration* of the real exchange rate.

# Consumption dynamics and RER determination

## Equilibrium

- Domestic market clearing and the household's intertemporal budget constraint then imply that  $c_t^T$  equals the annuity value of net foreign assets plus permanent income from the tradable endowment sequence:

$$c_t^T = \rho R a_t + \rho \sum_{s=0}^{\infty} R^{-s} w_{t+s}^T$$

- Recall that condition  $p_t = \frac{c_N(w^N, c_t^T)}{c_T(w^N, c_t^T)}$  pins down the *level* of the real exchange rate. Thus, as  $c_t^T \uparrow$ ,  $c_T(w^N, c_t^T) \downarrow$  and  $p_t \uparrow$  (i.e., a RER appreciation)

# RER determination and sectoral productivities

- Over longer horizons, factors of production can be reallocated between sectors, implying that tradable and non-tradable output are no longer exogenous
- This undermines the link between household wealth and the RER
- To see this, envisage a SOE that produces two goods, with CRS production functions
  - Tradable  $A_t^T f(K_t^T, L_t^T)$
  - Non-tradable  $A_t^N f(K_t^N, L_t^N)$
- Capital is internationally mobile, with world interest rate  $R_t$
- Labor is only mobile across sectors,  $L_t^T + L_t^N = 1$ , and earns  $w_t$
- Price of tradable normalized to one, relative price of non-tradable is  $p_t$

# RER determination and sectoral productivities

- If competitive domestic firms produce both goods, the marginal value products of the two factors in both sectors must equal the respective factor prices

$$\begin{aligned}A_t^T f_K(k_t^T, 1) + 1 - \delta &= R_t \\p_t A_t^N f_K(k_t^N, 1) + 1 - \delta &= R_t \\A_t^T f_L(k_t^T, 1) &= w_t \\p_t A_t^N f_L(k_t^N, 1) &= w_t\end{aligned}$$

- CRS imply that marginal products are a function of respective capital-labor ratios

# RER determination and sectoral productivities

- An increase in  $A_t^T$  will drive factors to the tradable sector
- Start with the zero-profit conditions for each sector:

$$\begin{aligned}A_t^T f(k_t^T, 1) &= w_t + (R_t - 1 + \delta)k_t^T \\ p_t A_t^N f(k_t^N, 1) &= w_t + (R_t - 1 + \delta)k_t^N\end{aligned}$$

- Take the total derivative w.r.t.  $A_t^T$ , and show the effect on  $p_t$

## RER determination and sectoral productivities

- Total derivative of  $A_t^T f(k_t^T, 1) = w_t + (R_t - 1 + \delta)k_t^T$  :

$$f(k_t^T, 1) + A_t^T \frac{df(k_t^T, 1)}{dk_t^T} \frac{dk_t^T}{dA_t^T} = \frac{dw_t}{dA_t^T} + (R_t - 1 + \delta) \frac{dk_t^T}{dA_t^T}$$

## RER determination and sectoral productivities

- Total derivative of  $A_t^T f(k_t^T, 1) = w_t + (R_t - 1 + \delta)k_t^T$  :

$$f(k_t^T, 1) + A_t^T \frac{df(k_t^T, 1)}{dk_t^T} \frac{dk_t^T}{dA_t^T} = \frac{dw_t}{dA_t^T} + (R_t - 1 + \delta) \frac{dk_t^T}{dA_t^T}$$

- Total derivative of  $p_t A_t^N f(k_t^N, 1) = w_t + (R_t - 1 + \delta)k_t^N$  :

$$\frac{dp_t}{dA_t^T} A_t^N f(k_t^N, 1) + p_t A_t^N \frac{df(k_t^N, 1)}{dk_t^N} \frac{dk_t^N}{dA_t^T} = \frac{dw_t}{dA_t^T} + (R_t - 1 + \delta) \frac{dk_t^N}{dA_t^T}$$



## RER determination and sectoral productivities

- Total derivative of  $A_t^T f(k_t^T, 1) = w_t + (R_t - 1 + \delta)k_t^T$  :

$$f(k_t^T, 1) + A_t^T \frac{df(k_t^T, 1)}{dk_t^T} \frac{dk_t^T}{dA_t^T} = \frac{dw_t}{dA_t^T} + (R_t - 1 + \delta) \frac{dk_t^T}{dA_t^T}$$

- Total derivative of  $p_t A_t^N f(k_t^N, 1) = w_t + (R_t - 1 + \delta)k_t^N$  :

$$\frac{dp_t}{dA_t^T} A_t^N f(k_t^N, 1) + p_t A_t^N \frac{df(k_t^N, 1)}{dk_t^N} \frac{dk_t^N}{dA_t^T} = \frac{dw_t}{dA_t^T} + (R_t - 1 + \delta) \frac{dk_t^N}{dA_t^T}$$

- Thus:

$$\frac{A_t^T}{p_t} \frac{dp_t}{dA_t^T} = \frac{A_t^T f(k_t^T, 1)}{p_t A_t^N f(k_t^N, 1)}$$

## RER determination and sectoral productivities

- Total derivative of  $A_t^T f(k_t^T, 1) = w_t + (R_t - 1 + \delta)k_t^T$  :

$$f(k_t^T, 1) + A_t^T \frac{df(k_t^T, 1)}{dk_t^T} \frac{dk_t^T}{dA_t^T} = \frac{dw_t}{dA_t^T} + (R_t - 1 + \delta) \frac{dk_t^T}{dA_t^T}$$

- Total derivative of  $p_t A_t^N f(k_t^N, 1) = w_t + (R_t - 1 + \delta)k_t^N$  :

$$\frac{dp_t}{dA_t^T} A_t^N f(k_t^N, 1) + p_t A_t^N \frac{df(k_t^N, 1)}{dk_t^N} \frac{dk_t^N}{dA_t^T} = \frac{dw_t}{dA_t^T} + (R_t - 1 + \delta) \frac{dk_t^N}{dA_t^T}$$

- Thus:

$$\frac{A_t^T}{p_t} \frac{dp_t}{dA_t^T} = \frac{A_t^T f(k_t^T, 1)}{p_t A_t^N f(k_t^N, 1)}$$

- An increase in non-tradable sector productivity decreases  $p_t$

# RER determination and sectoral productivities

- If both productivities increase, as long as  $A_t^T$  grows faster than  $A_t^N$ , and  $A_t^T f(k_t^T, 1) > p_t A_t^N f(k_t^N, 1)$ , then  $p_t$  goes up
- Empirically, this tends to be the case
- Given our previous result, countries with relatively high productivity growth in the tradable sector tend to experience a RER appreciation: This is known as the [Harrod-Balassa-Samuelson](#) effect
- If international income differences reflect differences in tradable sector productivity, the Harrod-Balassa-Samuelson effect predicts a positive relation between income and RER across countries