Macroeconomics III - Lecture 3

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Ramsey model

Laws of motion

- Laws of motion for capital and consumption:

$$\begin{aligned} k_{t+1} &= k_t(1-\delta) + f(k_t, 1) - c_t \\ u'(c_t) &= \beta(1 + f_K(k_{t+1}, 1) - \delta)u'(c_{t+1}) \end{aligned}$$

- Note that, given k_0, these equations pin down k_{t+1} and c_{t+1}, conditional on the initial value of consumption, c_0 (will get back to this later on)

Ramsey model

Analysis

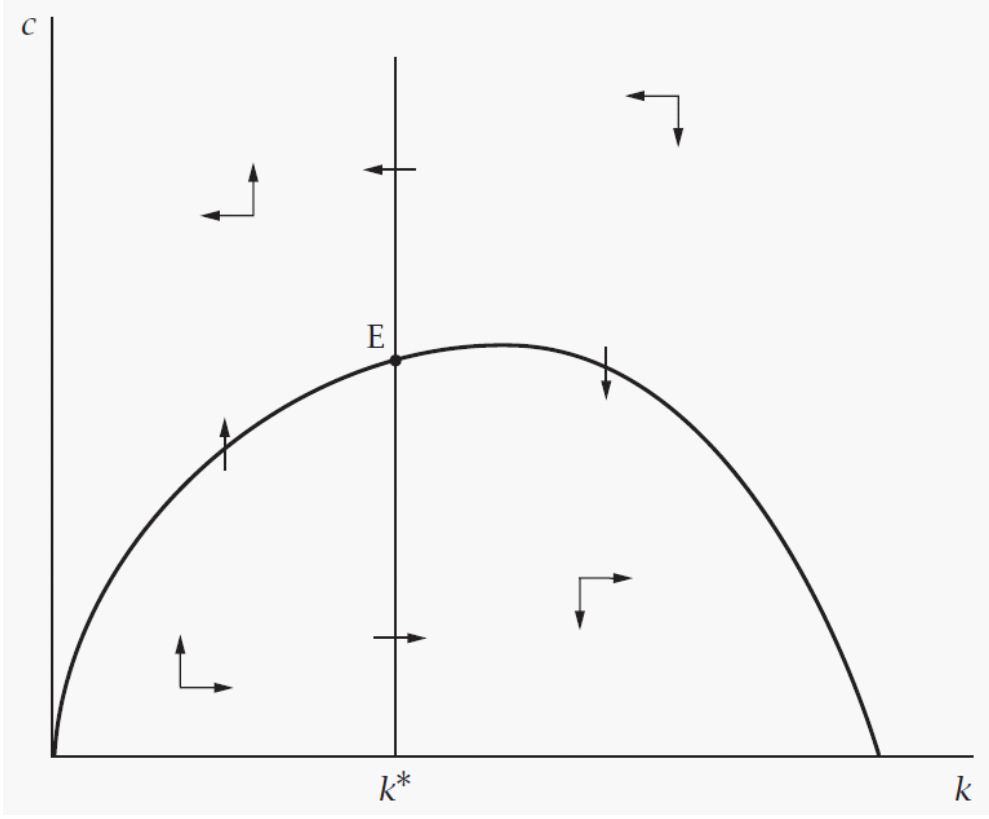
- Now we will perform a graphical analysis of the economy's dynamics
- To do this we plot in a k, c phase diagram the curves (*loci*) that correspond to $c_{t+1} = c_t = c$ and $k_{t+1} = k_t = k$, i.e. the combinations of k and c that respectively imply no time change for these variables:

$$\begin{aligned} c &= f(k,1) - \delta k \\ 1 &= \beta(1 + f_K(k,1) - \delta) \end{aligned}$$

- Their intersection defines the steady state, k^*, c^*. How do c and k move outside these curves? For any initial allocation, is the steady state always attained?

Ramsey model

Analysis



- We can combine both loci for the complete phase diagram
- There is a *balanced growth path (BGP)* at point **E**: c and k are constant at their steady-state level, c^* and k^*
- Arrows suggest that we may converge to BGP if we start somewhere in NE or SW quadrant

Government spending

- Let's talk about fiscal policy
- Should we engage in stimulus spending? What is the fiscal multiplier? Does output increase 1-for-1 with government spending, more than 1 (Keynes) or less than 1?
- To keep things simple, we will start with (and relax some of these later)
  - A government spends a fixed amount of resources each period (military, education, arts, sports...)
  - Finances these with lump sum taxes on households (no debt, no distortionary taxes)
  - Government spending is pure consumption ("thrown into the ocean": no effect on household utility, or firm production)

Government spending

- Household problem almost unchanged

$$a_{t+1} = a_t(1 + r_t - \delta) + w_t + z_t - c_t - T_t$$

- Firm problem unchanged
- New! Government budget constraint

$$T_t = G_t$$

Government spending

- How do the equilibrium conditions change?
- Law of motion for capital (derive this mechanically by imposing market clearing, i.e. combine household and government budget constraints with equilibrium prices)

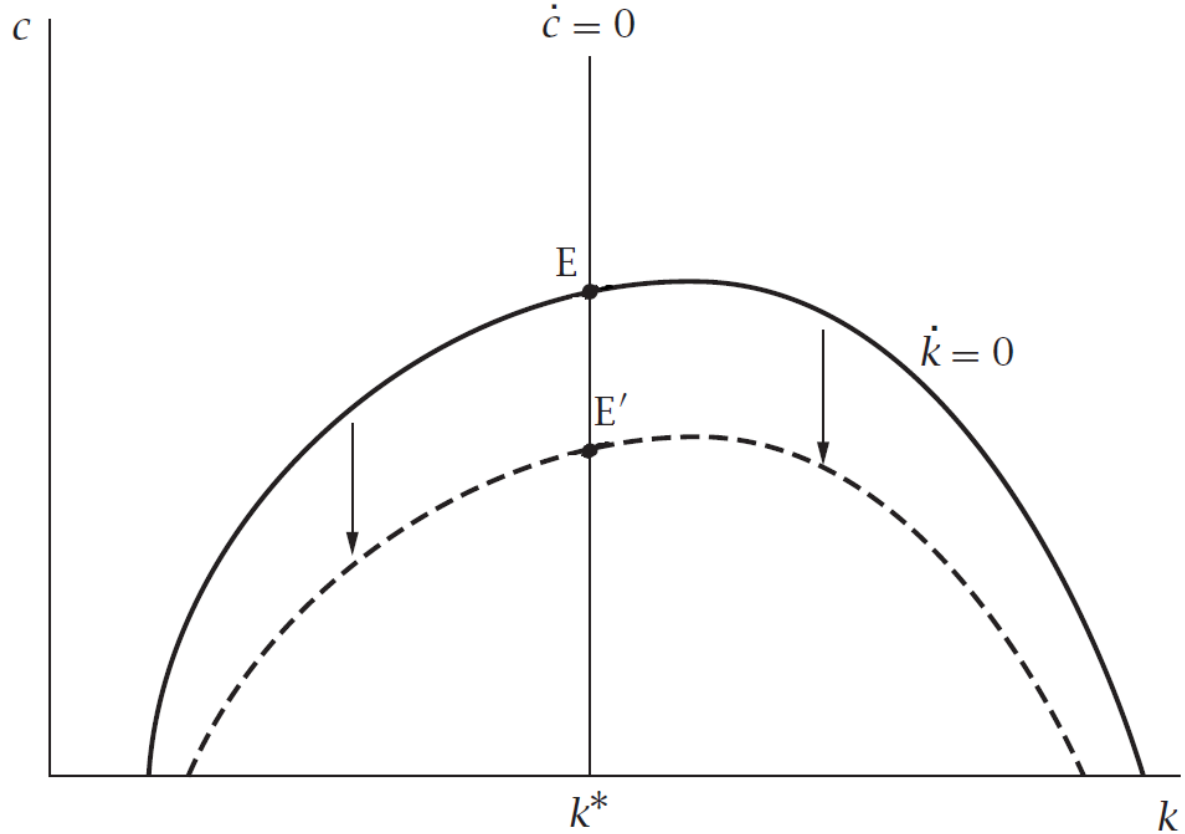
$$k_{t+1} = k_t(1-\delta) + f(k_t, 1) - c_t - G_t$$

  - Why? Government purchases here are just another form of consumption
- Euler: Unchanged

$$u'(c_t) = \beta(1 + r_{t+1} - \delta)u'(c_{t+1})$$

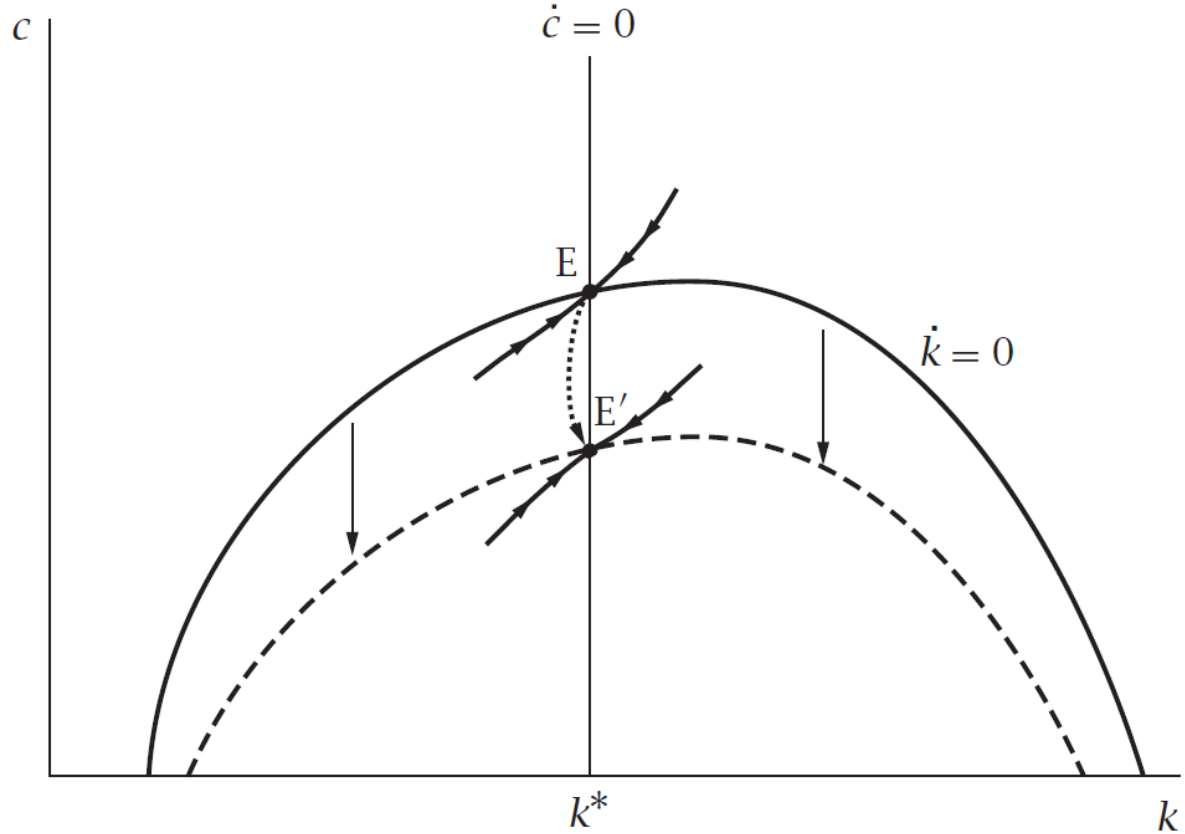
  - Why? Lump sum taxes are just another source of income. Doesn't change how you trade off consumption between today and tomorrow
  - Practice: Verify this yourselves
  - When could the Euler equation change?

Government spending graphically: The long run



- Solid: $G = 0$, dashed: $G > 0$
- For given k, $G \uparrow, c \downarrow$ to keep $\dot{k_{t+1}} = k_t$
- In the long run, (move from E to E') **public consumption replaces private consumption 1-for-1**
- Capital accumulation and output are not affected

Government spending graphically: The transition

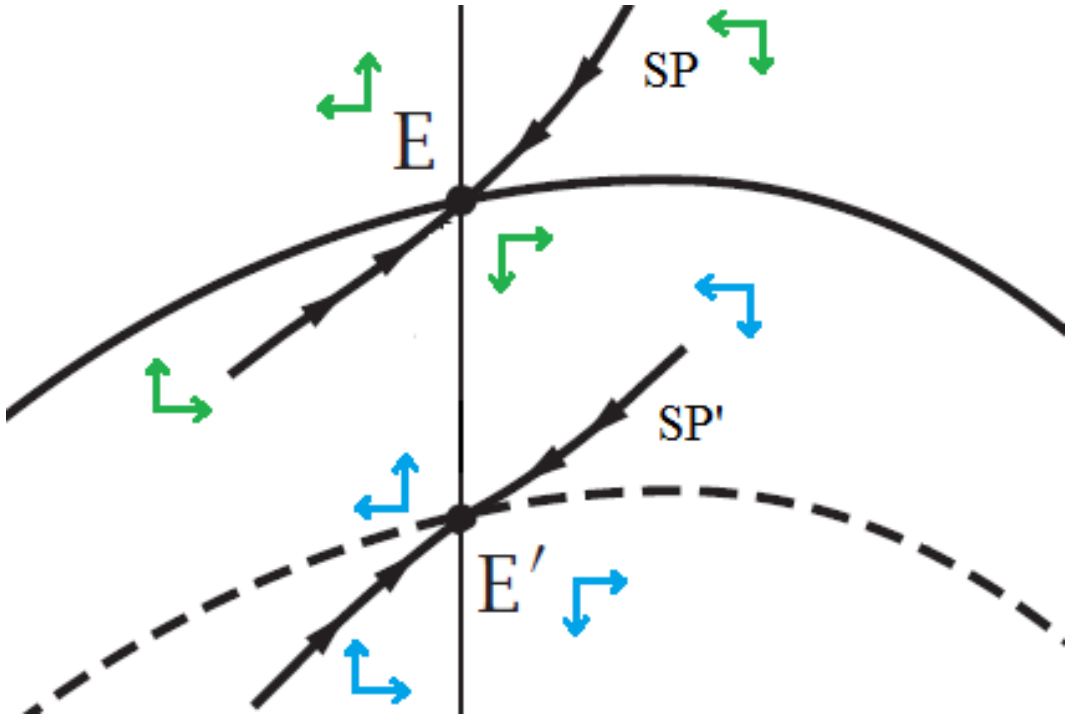


- Suppose $G = 0$ for $t < t_0$. At $t = t_0$, $G > 0$ unexpectedly and permanently
- Capital can't jump, and we must be on the new saddle path at t_0
- Thus, consumption adjust immediately to new BGP

Temporary shocks

- Realistically, government spending programs are not permanent
- How do Ramsey households respond if the shock does not last forever (but is unexpected)?

Temporary shocks



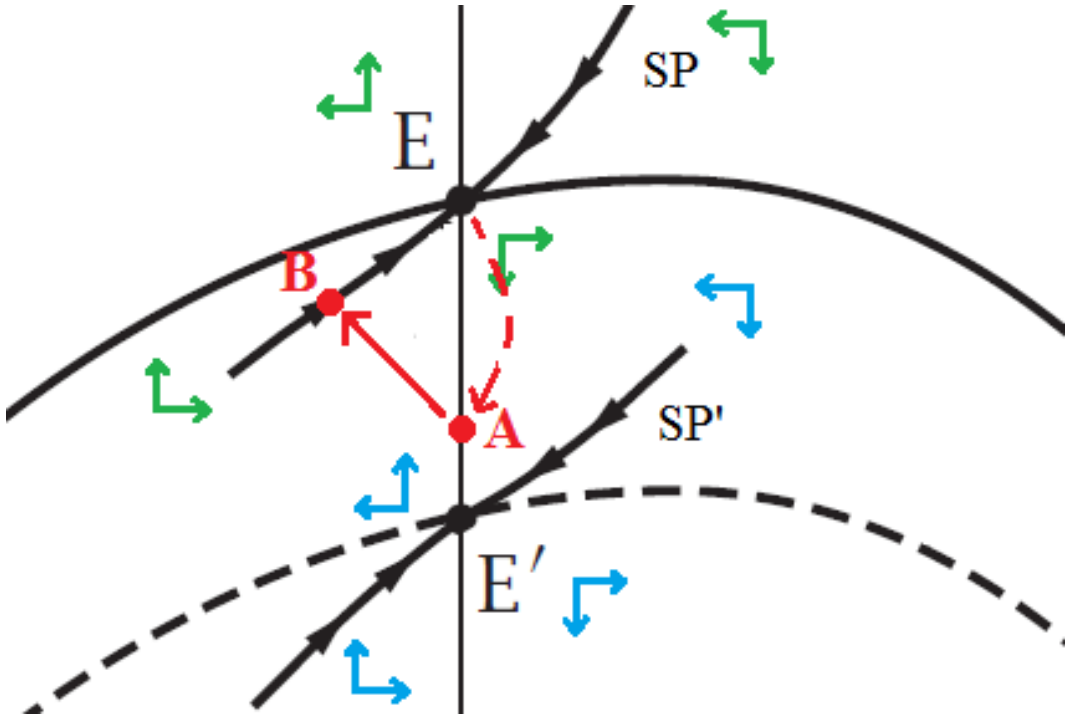
- Experiment: Start on BGP with $G = 0$. At t_0 we unexpectedly learn that for $t \in [t_0, t_1) : G > 0$, and for $t \geq t_1 : G = 0$
- How does the economy adjust?
  - Option 1: Stay put at E ?
  - Option 2: Jump to E' at t_0, and jump back at t_1?
  - Option 3: ?

Temporary shocks

- 3 key steps to figuring out the adjustment:

1. When does new information arrive?
   - The *only* time c can jump without violating Euler
   - Here: Only t_0. Not at t_1
2. Which equations govern the dynamics in the long run, and from when on?
   - Must be on correct saddle path at exactly that time to converge
   - Here, the original equations, from t_1 on
3. Which equations govern the dynamics between t_0 and t_1?
   - For $t \in [t_0, t1)$, the dynamics are governed by the new equations

Temporary shocks



- At t_0, jump to A
- From t_0 to t_1, dynamics governed by new system (blue), so drift NW (dynamics governed by blue arrows from t_0 to t_1)
- At t_1, arrive at B
- From t_1 on, dynamics are governed by old system (green), so converge back to E

Temporary shocks

- Households smooth consumption in response to temporary shocks:
  - They adjust current consumption less than the drop in current income
  - During the temporary income drop, they dissave (run down capital) to sustain higher consumption
  - Once their income goes back to normal, they replenish the capital stock
- This is optimal because we've assumed concavity in the utility function

Temporary shocks

- Temporary stimulus programs in the Ramsey model
  - Depress private consumption
  - "Crowd out" private investment (reduce the capital stock) and
  - Reduce output
- Reason:
  - The government budget constraint holds: Higher spending means higher taxes and so less income for households
  - Temporary shock
  - Consumption-smoothing households

Anticipated shocks

- Examples: Olympics, planned expiry of longstanding government programs

Practice for you:

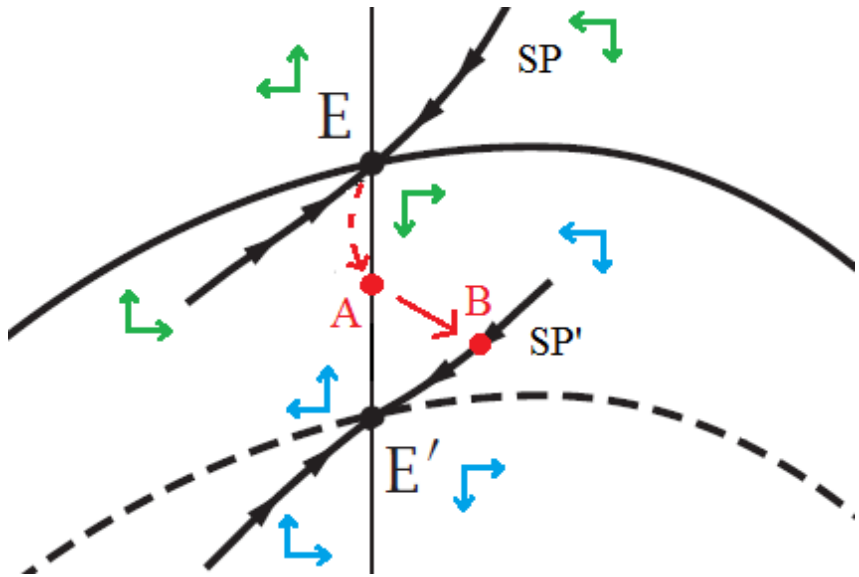
- Experiment: Start at BGP with $G = 0$. At t_0, we unexpectedly learn that for $t \geq t_1 : G > 0$, and for $t \in [t_0, t_1), G = 0$
- Draw the adjustment in the phase diagram (follow the 3 steps as in slide 9)
- What is the intuition from the households' perspective?

Anticipated shocks

- 3 key steps to figuring out the adjustment to this shock:

1. When does new information arrive?
   - Only time c can jump
   - Here: Only t_0. Not at t_1
2. Which equations govern the dynamics in the long run, and from when on?
   - Must be on correct saddle path at exactly that time to converge
   - Here, the *new* equations from t_1 on
3. Which equations govern the dynamics between t_0 and t_1?
   - For $t \in [t_0, t1)$, the dynamics are governed by the *old* equations

Permanent increase in G, announced t_0, implemented t_1



- Consumption jumps to A at t_0
- Must be on saddle path (e.g. at B) at t_1 to converge to E'
- Old (green) dynamics take the economy from A to B
- Economics: Households expect lower income in the future, so it is optimal to start adjusting consumption downwards now. This implies temporarily higher capital accumulation and output

Government debt

- We have seen that balanced budget stimulus spending does not work in the Ramsey model
  - Permanent shock: Private consumption falls 1-for-1 with increases in government spending, no effect on output and capital accumulation
  - Temporary shock: Capital and output fall temporarily, consumption also (but by less)
- Reason: Government expenditures had to be paid for with taxes, and households adjust consumption path in response to drops in after tax income
- So, natural question is: **Is stimulus spending more successful if we finance it with debt instead of taxes?**

Government debt: Budget constraints

- Suppose the government can borrow from the private sector
- Then there are 2 ways to finance an increase in expenditures - taxes T_t or debt b_t - and its budget constraint becomes

$$b_{t+1} = G_t - T_t + R_t b_t \tag{1}$$

  Note that all variables here are in per capita terms!
- Government revenue: Debt issuance $b_{t+1} - b_t$ and taxes T_t
- Government expenditures: Spending G_t and interest expenses $r_t b_t$

Government debt: Households

- Do households change their behavior when governments borrow instead of raising taxes to finance expenditures?
- We assume that households hold the government debt (in reality, through pension plans and mutual funds, for example)
- The household budget constraint is (assume zero depreciation, without loss of generality)

$$a_{t+1} = R_t a_t + w_t + z_t - c_t - T_t$$

with assets now given by

$$a_t = k_t + b_t$$

Ricardian equivalence I

- To check how HH behavior and the equilibrium are affected by a government borrowing, instead of raising taxes, let's consider the equilibrium conditions
- Do HHs change how they allocate consumption across time?
  - No. Euler equation the same whether government runs balanced budget or borrows (can you show this?)
- TVC clearly unchanged
- Firm behavior not affected

Ricardian equivalence I

- Do the total resources in the economy change?
  - No. The equilibrium LOM for capital only depends on expenditures, not debt or taxes. Substitute GBC into HHBC to see this:

$$\triangle k_{t+1} + \triangle b_{t+1} = (k_t + b_t)r_t + w_t + z_t - c_t - T_t$$

$$\triangle b_{t+1} = G_t - T_t + r_t b_t$$

$$\triangle k_{t+1} = f(k_t, 1) - c_t - G_t$$

- **Key result (Ricardian equivalence): For a given path of government expenditures, whether they are financed with lump sum taxes or debt does not affect the equilibrium allocation**

Ricardian equivalence II

- Intuition for Ricardian equivalence?
- Households know that any government expenditures have to eventually be paid for by taxes
- In response to higher government spending financed with debt households reduce consumption and save in anticipation of the future tax hike
- They save exactly as much as the government needs to borrow
- The effect on capital accumulation and output is the same regardless of the finance method

Ricardian equivalence III

- Important assumptions for Ricardian equivalence to hold
  - Lump-sum taxes
  - Infinitely-lived households
  - Closed economy, no international investors
  - No default risk
  - Unproductive government spending

Distortionary taxation

- Taxes are distortionary when they affect optimal decisions
- Lump sum taxes: Not distortionary
- Proportional taxes: Distortionary
- Examples of proportional taxes: labor income taxes, consumption taxes (VAT), capital income taxes
- Key result: Distortionary capital taxation reduces equilibrium capital accumulation and welfare

Households and the government

- Let τ_t denote the tax rate on capital income
- Household budget constraint

$$\triangle a_{t+1} = a_t(1 - \tau_t)r_t + w_t + z_t - c_t$$

- Assume for simplicity that the government rebates any tax revenue to households so its budget constraint is

$$T_t = \tau_t r_t a_t$$

- Households are price takers: take τ, T, w, and r as given, when making their decisions

Firms and equilibrium LOM for capital

- The firm problem is not affected - they continue to rent capital and labor, and optimally pay both their marginal product (check it)
- Combine household and government budget constraints, and use equilibrium prices to find

$$\triangle\ k_{t+1} = f(k_t, 1) - c_t - G_t$$

- *The equilibrium law of motion for capital is unaffected by capital taxes*

Euler equation

- Let $\tilde{r}_t \equiv (1 - \tau_t) r_t$ the after tax interest rate
- Then the Euler equation is given by

$$u'(c_t) = \beta(1 + \tilde{r}_{t+1}) u'(c_{t+1})$$

- *Capital taxes affect optimal consumption growth: The higher the tax, the lower the incentive to save, the slower consumption growth*
- Mechanically: The after-tax return to capital $1 + \tilde{r}$ must still be equal to the discount rate $1/\beta$ on the BGP. So the pre-tax return is higher, and k^* lower

A thought experiment

- There are large cross-country differences in tax rate on capital returns. Why is that? Wouldn't everybody want to invest in the lowest tax country?

A thought experiment

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- This effectively asks: Suppose you live in a high capital tax country. Preferences across countries are identical. Do you have an incentive to invest in a neighboring low capital tax country in the long run?

A thought experiment

- There are large cross-country differences in tax rate on capital returns. Why is that? Wouldn't everybody want to invest in the lowest tax country?
- This effectively asks: Suppose you live in a high capital tax country. Preferences across countries are identical. Do you have an incentive to invest in a neighboring low capital tax country in the long run?
- Answer: No, the after-tax return on capital will be the same across countries. (There will be more investment in the low tax country, to the point where the after tax returns are equalized)