Macroeconomics III

Lecture 5

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Government spending

- What happens to capital accumulation when we introduce a government that taxes households and spends resources?
- We asked this in the infinite horizon context, instructive to see the effect of population turnover
- Stick with simplest case again: Government makes “useless” purchases G_t (per worker) and pays for them by levying lump sum taxes T_t (per worker) *on the young* each period

Government spending

- Agents' budget constraints when young and old become

$$\begin{aligned} c_{1t} + s_t &= w_t - T_t \\ c_{2t+1} &= (1 + r_{t+1})s_t \end{aligned}$$

- The only place T_t enters is in wage income when young, redefine $\tilde{w}_t \equiv w_t - T_t$, then we know (from log-utility)

$$s_t = \frac{1}{2+\rho}\tilde{w}_t$$

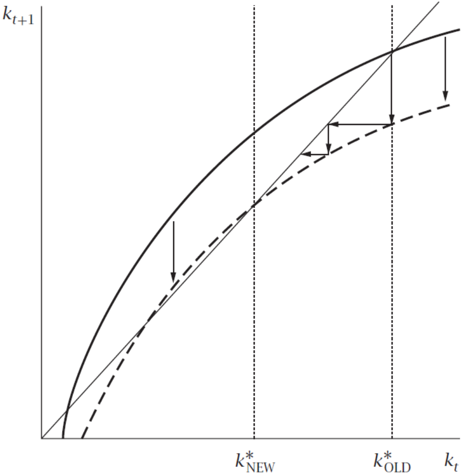
and

$$k_{t+1} = \frac{s_t}{1+n} = \frac{\tilde{w}_t}{(1+n)(2+\rho)} = \frac{(1-\alpha)k_t^{\alpha} - T_t}{(1+n)(2+\rho)}$$

- In equilibrium:

$$k_{t+1} = \frac{(1-\alpha)k_t^{\alpha} - G_t}{(1+n)(2+\rho)}$$

Effect of permanent unexpected increase in government spending



$$k_{t+1} = \frac{(1-\alpha)k_t^{\alpha} - G_t}{(1+n)(2+\rho)}$$

- Capital falls! Very different from Ramsey
- Why....?

Effect of permanent unexpected increase in government spending

- In Ramsey, $G > 0$ had no effect on capital accumulation
  - Permanent drop in PDV of lifetime income, thus can't smooth effect on consumption and adjust 1:1
- In OLG:
  - As in Ramsey, households are poorer in PDV terms so they need to reduce PDV consumption
  - Euler equation: Optimal to smooth, ie reduce *both* c_{1t} and c_{2t+1}
  - The *only* way to lower c_{2t+1}, is to lower s_t
- Underlying reason: Only the young pay taxes

Government spending: Other types of shocks

- Let's consider other types of shocks
- Does stimulus spending work here (i.e., a temporary unexpected increase in spending)?

- How about anticipated shocks?

Government spending: Other types of shocks

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- Does stimulus spending work here (i.e., a temporary unexpected increase in spending)?
  - Same effect as for permanent shock
  - Young individuals today don't care if G_t is higher just today or forever, they behave exactly the same
  - Would be more similar to Ramsey if we had more than 2 periods of life
- How about anticipated shocks?

Government spending: Other types of shocks

- Let's consider other types of shocks
- Does stimulus spending work here (i.e., a temporary unexpected increase in spending)?
  - Same effect as for permanent shock
  - Young individuals today don't care if G_t is higher just today or forever, they behave exactly the same
  - Would be more similar to Ramsey if we had more than 2 periods of life
- How about anticipated shocks?
  - No effect
  - Young individuals today don't care if G_t is announced to be higher tomorrow, they only care about G_t today
  - Would be more similar to Ramsey if we had more than 2 periods of life

Social security and gov. debt

- Two policy applications of OLG models:

1. Social security (BF 3.2)
   1.1 Fully funded system
   1.2 PAYG system
2. Government debt
   2.1 Ricardian equivalence
   2.2 Equivalence between social security and government debt

Social Security

- Social security: Government provides for citizens who don't have an income of their own
- Motivations: Retirement, short-sightedness/insufficient own savings, redistribution
- Broadly, 2 types of social security systems:

1. Unfunded pay-as-you-go (PAYG) systems
   - Current young pay contributions as benefits *to current old*
2. Fully funded systems
   - Current young pay contributions as benefits *to themselves when old*

- Since social security has to do with savings, it can affect aggregate capital accumulation - let's see exactly how

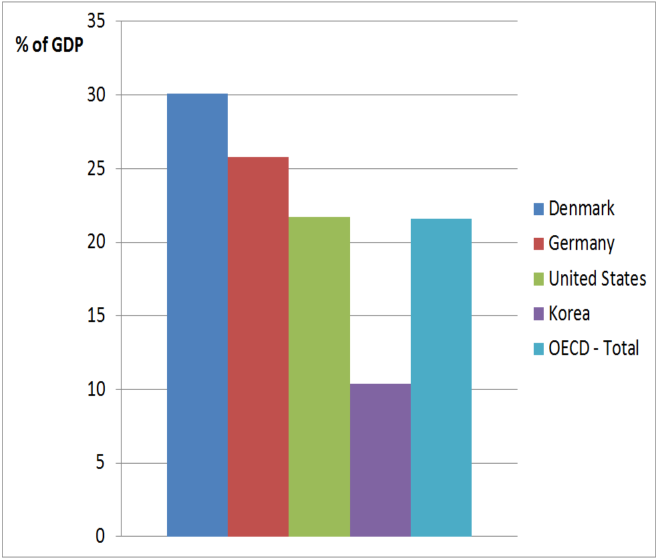
Social Security around the world

- Social security programs exist in most countries around the world
  - Germany: First country to introduce social security (Bismarck in 1884)
  - Widely adopted in developed countries post WWI
  - US: Introduced by Roosevelt in 1935 after the Great Depression
  - Denmark and Scandinavian welfare states
- All OECD systems are PAYG, fully funded more common in developing countries

Social Security benefits

- Danish retiree receives >80% of earnings as social security benefits
- US retiree only 40%!

Public Social Security expenditures



- Old age is the largest program ($\approx$ 10% of GDP), next largest health

Social Security issues

- Many PAYG systems have funding problems because of aging populations ($n < 0$)
  - In the US in 1935, 45 people paid into the system for every retiree. Today: 3:1!!!
- Key issue when labor is mobile - cf. migration debate in Europe: Should recent migrants be able to receive benefits in destination country, without having paid in when young themselves?
- Social security can distort incentives
  - E.g., encourages early retirement, discourages to work. Policies to counteract: Later retirement ages, benefits tied to contributions, flexible labor markets

Social security and gov. debt

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Fully funded system

- Government raises contributions d_t from the young, invests these and pays them out with interest as benefits next period:

$$b_{t+1} = (1 + r_{t+1})d_t$$

- Budget constraint when young in t

$$c_{1t} + s_t + d_t = w_t$$

- Budget constraint when old in $t + 1$

$$\begin{aligned} c_{2t+1} &= (1 + r_{t+1})s_t + b_{t+1} \\ & (1 + r_{t+1})(s_t + d_t) \end{aligned}$$

Same return on contributions and individual savings, as physical capital is the only means of saving (also for the pension system)

Fully funded system

- Agents do not choose d_t, but take it as given, so they perceive it as a reduction in income - just like lump sum taxes T_t
- Hence their Euler equation is unchanged:

$$u'(c_{1t}) = \frac{1 + r_{t+1}}{1 + \rho} u'(c_{2t+1})$$

and, using the budget constraints:

$$u'\left[w_t - (s_t + d_t)\right] = \frac{1 + r_{t+1}}{1 + \rho} u'\left[(1 + r_{t+1})(s_t + d_t)\right]$$

- Aggregate savings (capital accumulation) in t are now the sum of individual savings and SS contributions

$$s_t + d_t = (1 + n)k_{t+1}$$

- Effect of fully-funded social security on capital accumulation?

Fully funded system: Effect on capital accumulation

- Define $\tilde{s} \equiv s + d$, compare the equations characterizing the equilibrium in the two economies, with and without SS:

$$c_{1t} + \tilde{s}_t = w_t \qquad c_{1t} + s_t = w_t$$

$$c_{2t+1} = (1 + r_{t+1})\tilde{s}_t \qquad c_{2t+1} = (1 + r_{t+1})s_t$$

$$u'(c_{1t}) = \frac{1 + r_{t+1}}{1 + \rho} u'(c_{2t+1}) \qquad u'(c_{1t}) = \frac{1 + r_{t+1}}{1 + \rho} u'(c_{2t+1})$$

$$k_{t+1} = \frac{\tilde{s}_t}{1 + n} \qquad k_{t+1} = \frac{s_t}{1 + n}$$

- Any k_t that solves the LHS system also solves the RHS
- **No effect on capital accumulation**
- Public savings d_t exactly offset private savings s_t - they have the same return, after all

Social security and gov. debt

- Two policy applications of OLG models:

1. Social security (BF 3.2)
   1.1 Fully funded system
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PAYG system

- In PAYG system, government raises contributions d_t from current young, and pays them out as benefits to ***current*** old:

$$b_t = (1+n)d_t$$

- Budget constraint when young in t

$$c_{1t} + s_t + d_t = w_t$$

- Budget constraint when old in $t+1$

$$c_{2t+1} = (1+r_{t+1})s_t + (1+n)d_{t+1}$$

Implicit return on contributions is population growth - there are more young paying in today than old recipients

PAYG system

- Contributions are taken as given, so Euler equation unchanged
- Substitute out for consumption, using the budget constraints, we get:

$$u'\left[w_t - (s_t + d_t)\right] = \frac{1 + r_{t+1}}{1 + \rho} u'\left[(1 + r_{t+1})s_t + (1 + n)d_{t+1}\right]$$

- Aggregate savings are given by

$$s_t = (1 + n)k_{t+1}$$

Note that the contributions are *not* transferred across time in this system, so they don't contribute to capital accumulation

PAYG system: Effect on capital accumulation

- Is k_{t+1} affected now?
- Let's compare equilibrum conditions with and without SS:

$$c_{1t} + s_t + d_t = w_t \qquad\qquad c_{1t} + s_t = w_t$$

$$c_{2t+1} = (1+r_{t+1})s_t + (1+n)d_{t+1} \qquad\qquad c_{2t+1} = (1+r_{t+1})s_t$$

$$u'(c_{1t}) = \frac{1+r_{t+1}}{1+\rho}u'(c_{2t+1}) \qquad\qquad u'(c_{1t}) = \frac{1+r_{t+1}}{1+\rho}u'(c_{2t+1})$$

$$k_{t+1} = \frac{s_t}{1+n} \qquad\qquad k_{t+1} = \frac{s_t}{1+n}$$

- The same k_t won't solve both systems of equations in general
- So PAYG systems do affect capital accumulation. How?

PAYG system: Special case and analytical solution

- Let's use the log CD case to identify the sign of the effect. Assume:

$$u(c) = \log c$$

and

$$f(k) = k^{\alpha}$$

- Also assume full depreciation

$$\delta = 1$$

such that $R = r$ (in general, $R = 1 + f'(k) - \delta$).

- Finally, suppose that contributions are a fixed fraction τ of wage income:

$$d_t = \tau w_t$$

PAYG system: Special case and analytical solution

- Let's derive a closed form expression for individual savings

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PAYG system: Special case and analytical solution

- Let's derive a closed form expression for individual savings

$$u'(c_{1t}) = \frac{r_{t+1}}{1+\rho} u'(c_{2t+1})$$
$$\frac{1}{c_{1t}} = \frac{r_{t+1}}{1+\rho} \frac{1}{c_{2t+1}}$$

PAYG system: Special case and analytical solution

- Let's derive a closed form expression for individual savings

$$u'(c_{1t}) = \frac{r_{t+1}}{1+\rho} u'(c_{2t+1})$$

$$\frac{1}{c_{1t}} = \frac{r_{t+1}}{1+\rho} \frac{1}{c_{2t+1}}$$

$$\frac{1}{w_t - s_t - d_t} = \frac{r_{t+1}}{1+\rho} \frac{1}{r_{t+1} s_t + (1+n) d_{t+1}}$$

PAYG system: Special case and analytical solution

- Let's derive a closed form expression for individual savings

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\begin{aligned}
u'(c_{1t}) &= \frac{r_{t+1}}{1+\rho} u'(c_{2t+1}) \\
\frac{1}{c_{1t}} &= \frac{r_{t+1}}{1+\rho} \frac{1}{c_{2t+1}} \\
\frac{1}{w_t - s_t - d_t} &= \frac{r_{t+1}}{1+\rho} \frac{1}{r_{t+1} s_t + (1+n) d_{t+1}} \\
r_{t+1} s_t + (1+n) d_{t+1} &= \frac{r_{t+1}}{1+\rho} (w_t - s_t - d_t)
\end{aligned}
$$

PAYG system: Special case and analytical solution

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\frac{1}{w_t - s_t - d_t} &= \frac{r_{t+1}}{1+\rho} \frac{1}{r_{t+1} s_t + (1+n) d_{t+1}} \\
r_{t+1} s_t + (1+n) d_{t+1} &= \frac{r_{t+1}}{1+\rho} (w_t - s_t - d_t)
\end{aligned}
$$

- Re-arrange to get

$$
s_t = \frac{1}{2+\rho} \left[(w_t - d_t) - \frac{1+\rho}{r_{t+1}} (1+n) d_{t+1} \right]
$$

PAYG system: Special case and analytical solution

- Optimal individual savings:

$$s_t = \frac{1}{2+\rho}\left[(w_t - d_t) - \frac{1+\rho}{r_{t+1}}(1+n)d_{t+1}\right]$$

- Savings are a constant share of: disponsable income when young minus the current value of gross SS benefits when old
- *Ceteris paribus*, young households save ***less*** because
  - They have less disposable income when young and
  - They know they'll get benefits only when already old

PAYG system: Special case and analytical solution

- Let's derive the equilibrium expression for k_{t+1}

PAYG system: Special case and analytical solution

- Let's derive the equilibrium expression for k_{t+1}
- Using s_t we find

$$k_{t+1} = \frac{1}{1+n} s_t = \frac{1}{(1+n)(2+\rho)}(w_t - d_t) - \frac{1+\rho}{2+\rho}\frac{1}{r_{t+1}} d_{t+1}$$

PAYG system: Special case and analytical solution

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- Since $d_t = \tau w_t$

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)} w_t(1-\tau) - \frac{1+\rho}{2+\rho}\frac{1}{r_{t+1}} \tau w_{t+1}$$

PAYG system: Special case and analytical solution

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$$k_{t+1} = \frac{1}{(1+n)(2+\rho)} w_t (1-\tau) - \frac{1+\rho}{2+\rho}\frac{1}{r_{t+1}} \tau w_{t+1}$$

- Using equilibrium $\frac{w_{t+1}}{r_{t+1}} = \frac{(1-\alpha)k_{t+1}^{\alpha}}{\alpha k_{t+1}^{\alpha-1}}$

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)} w_t (1-\tau) - \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha} \tau k_{t+1}$$

PAYG system: Special case and analytical solution

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- Using equilibrium $\frac{w_{t+1}}{r_{t+1}} = \frac{(1-\alpha)k_{t+1}^{\alpha}}{\alpha k_{t+1}^{\alpha-1}}$

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)}w_t(1-\tau) - \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}\tau k_{t+1}$$

- Rearranging

$$k_{t+1} = \frac{1}{1+\frac{1+\rho}{2+\rho}\frac{(1-\alpha)}{\alpha}\tau}\left(\frac{1}{(1+n)(2+\rho)}(1-\tau)w_t\right)$$

PAYG system: Special case and analytical solution

$$k_{t+1} = \frac{1}{1 + \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}\tau}\left(\frac{1}{(1+n)(2+\rho)}(1-\tau)w_t\right)$$

- **PAYG social security lowers capital accumulation** because
  - There is less income when young and
  - You receive income when old
- Note PAYG SS can be used to restore dynamic efficiency
- Practice: Derive the analogous expressions for s_t and k_{t+1} for the fully funded system and show that that system has no effect on capital accumulation

Social security and gov. debt

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Government debt

- Governments can borrow and temporarily reduce taxes to finance expenditures
- Recall Ramsey: For a given sequence of expenditures, the financing mix did not matter
- Is that true here too - **does Ricardian equivalence hold in OLG**?

OLG with government debt

- Assume the government levies lump sum taxes and sells bonds to service outstanding debt. No government expenditures
- Government budget constraint is (go back to the original setting that allows for $\delta \in [0,1]$)

$$L_t T_t + B_{t+1} \quad = \quad R_t B_t$$

or in per capita terms,

$$T_t + (1+n) b_{t+1} \quad = \quad R_t b_t$$

- Aggregate savings? Split between capital and bonds

$$(k_{t+1} + b_{t+1})(1+n) = s_t$$

Household problem

- Households solve

$$\max u(c_{1t}) + \frac{1}{1+\rho} u(c_{2t+1})$$

$$\begin{aligned} c_{1t} + s_t &= w_t - T_t \\ c_{2t+1} &= R_{t+1} s_t \end{aligned}$$

- Assuming log utility, we can derive the Euler equation and hence savings function as before:

$$s_t = \frac{1}{2+\rho}(w_t - T_t)$$

- Household behavior does not depend on the composition of assets

Capital accumulation

- Equilibrium capital accumulation is given by

$$\begin{aligned} k_{t+1} &= \frac{1}{(1+n)(2+\rho)}[w_t - T_t] - b_{t+1} \\ &= \frac{1}{(1+n)(2+\rho)}[w_t - b_t R_t + (1+n)b_{t+1}] - b_{t+1} \end{aligned}$$

- Capital accumulation is lower with government debt for 2 reasons:
  1. Bonds replace capital and
  2. Bonds require taxes to cover interest payments
- The choice of government finance matters for capital accumulation
- If $b_t = b, \forall t$, then $T_t = b(r_t - n - \delta)$. If the economy is dynamically efficient ($r_t > n + \delta$), then $T_t > 0$

Ricardian equivalence

- So: equilibrium capital accumulation is not the same regardless of whether the government uses tax or deficit finance for a given sequence of expenditures
- In Ramsey, agents live forever and know that tax cuts today have to be reversed at some point in the future
  - Hence they don't spend windfall from tax cuts, they just save it - no effect on allocations of tax versus deficit finance
- In OLG, the young see a tax cut as a permanent increase in lifetime income, so they do adjust consumption
- **Even though the government will have to retire the debt and raise taxes eventually in OLG too, this tax hike falls on a *future* generation**

- Altruism/ bequest motives can restore Ricardian equivalence result

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Equivalence between debt and social security

- We will now show that (under specific assumptions) social security systems and government debt operate in a similar way
- Assume
  1. Log utility and CD production again
  2. Full depreciation
  3. Debt is a constant fraction of the capital stock $b_t = \gamma k_t$
- Then we can obtain a closed form expression for s_t and k_{t+1} - same steps as in Social Security example

Equivalence between debt and social security

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)}[w_t - b_t r_t - (1+n)b_{t+1}] - b_{t+1}$$

Equivalence between debt and social security

$$
\begin{aligned}
k_{t+1} &= \frac{1}{(1+n)(2+\rho)}[w_t - b_t r_t - (1+n)b_{t+1}] - b_{t+1} \\
&= \frac{1}{(1+n)(2+\rho)}[w_t - \gamma k_t(\alpha k_t^{\alpha-1}) - (1+n)\gamma k_{t+1}] - \gamma k_{t+1}
\end{aligned}
$$

Equivalence between debt and social security

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Equivalence between debt and social security

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&= \frac{1}{(1+n)(2+\rho)}[w_t - \gamma \alpha k_t^{\alpha}] - \frac{1}{2+\rho}\gamma k_{t+1} - \gamma k_{t+1} \\
&= \frac{1}{(1+n)(2+\rho)}\left(w_t - \gamma \frac{\alpha}{1-\alpha} w_t\right) - \frac{1+\rho}{2+\rho}\gamma k_{t+1}
\end{aligned}
$$

Equivalence between debt and social security

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\begin{aligned}
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&= \frac{1}{(1+n)(2+\rho)}[w_t - \gamma \alpha k_t^{\alpha}] - \frac{1}{2+\rho}\gamma k_{t+1} - \gamma k_{t+1} \\
&= \frac{1}{(1+n)(2+\rho)}\left(w_t - \gamma\frac{\alpha}{1-\alpha}w_t\right) - \frac{1+\rho}{2+\rho}\gamma k_{t+1} \\
&= \frac{1}{1+\frac{1+\rho}{2+\rho}\gamma}\left(\frac{1}{(1+n)(2+\rho)}\left(1-\gamma\frac{\alpha}{1-\alpha}\right)w_t\right)
\end{aligned}
$$

Equivalence between debt and social security

- With **government debt**

$$k_{t+1} = \frac{1}{1+\frac{1+\rho}{2+\rho}\gamma}\left(\frac{1}{(1+n)(2+\rho)}\left(1-\gamma\frac{\alpha}{1-\alpha}\right)w_t\right)$$

- With **PAYG social security**

$$k_{t+1} = \frac{1}{1+\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}\tau}\left(\frac{1}{(1+n)(2+\rho)}(1-\tau)w_t\right)$$

- These expressions are equivalent for

$$\gamma\frac{\alpha}{1-\alpha} = \tau$$

Equivalence between debt and social security

- Government debt can have the same effects as PAYG social security
  - Social security lowers savings due to contributions and benefits
  - Debt crowds out capital due to new issuance and debt service