Macroeconomics III

Lecture 6

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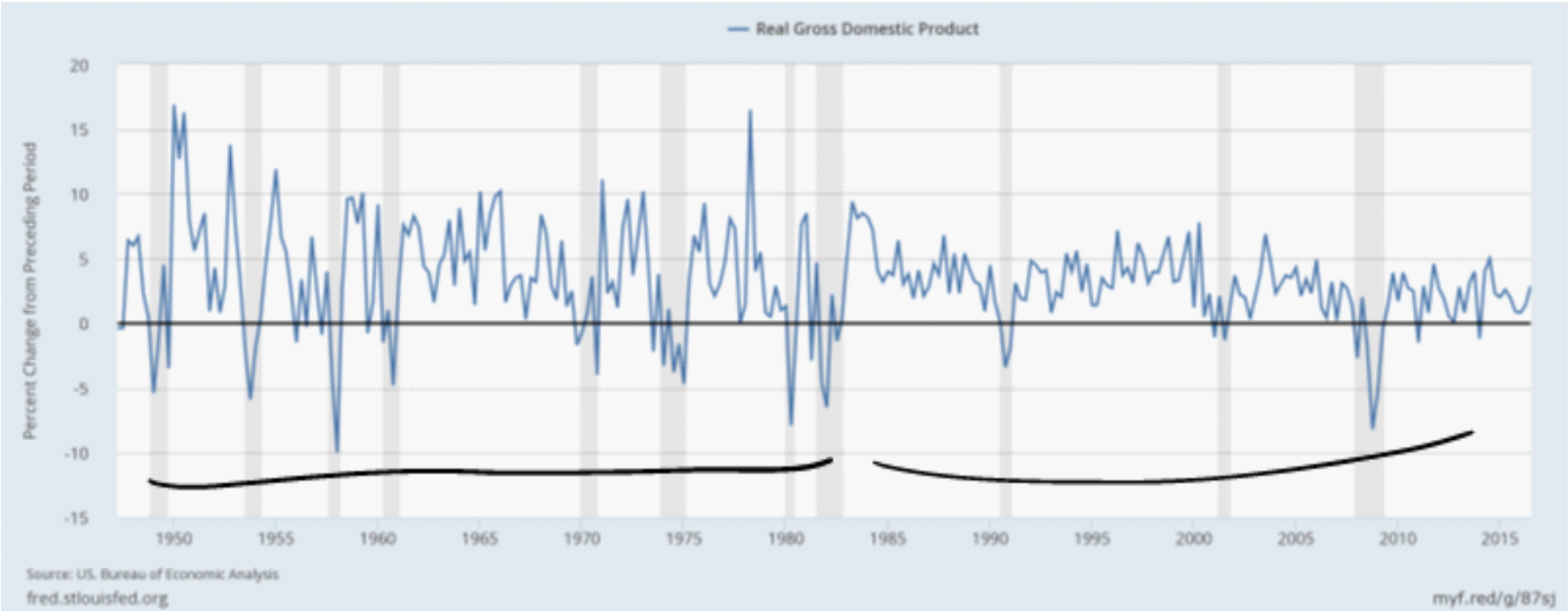
Real GDP fluctuations



Business cycles

As stressed by Lucas (1977), the finding that "business cycles are all alike" suggests that the nature of macroeconomic fluctuations does not hinge on institutional factors or country-specific idiosyncrasies, so that one can hope to construct a unified theory of the business cycle.

Real business cycles

Key insights

- Equilibrium approach to business cycle fluctuations, defined as a set of properties concerning comovements and persistence of main macroeconomic aggregates
- Unifying theory of growth and fluctuations (Ramsey model)
- Focus on the propagation mechanisms (especially over time) of shocks relies on intertemporal substitution effects
- Technological shocks as the main driving force of business cycle fluctuations

RBC = RAMSEY + STOCHASTIC ELEMENTS

↓

TECHNOLOGY

Real business cycles

Analytical features

- Reference model: neoclassical model of growth with uncertainty in the rate of technological progress
- Fluctuations are the aggregate result of the behavioral rules of rationally optimizing agents in a stochastic environment, subject to resource constraints
- Empirical methodology: calibration, rather than traditional econometric testing

Model Economy

1. Closed economy
2. The public sector is ignored
3. Four markets:
    1. Goods market: all-purpose output goods
    2. Labor market
    3. Rental market for capital goods
    4. Bond market
4. Perfect competition in all markets (prices are taken as given by producers)

OWNERS OF FACTORS OF PRODUCTION

General equilibrium: a quick overview

- Supply side:
  - In every period t, the individual firm employs labor at the market wage w_t and rents capital goods at the rental rate R_t
  - The combination of labour and capital produces the homogeneous output good. This can be used for consumption, as well as for investment
- Demand side:
  - Rational expectations → FORWARD LOOKING
  - Since households understand precisely how the economy works, they make the best possible prediction about the future path of wages and interest rates (to be developed further...)

DECENTRALIZED

EQUILIBRIUM

Technology

- Gross production is assumed to depend on a production technology that combines labor (N_t) and capital (K_t) inputs:

$$Y_t = A_t F(K_t, N_t)$$

where $F(\cdot)$ is a neoclassical production function

- To capture the upward trend in output per capita, the basic neoclassical model incorporates secular improvement in factor productivity

$$X_t = \gamma X_{t-1} \qquad \gamma > 1$$

- The standard Inada conditions apply:
  - $F(0) = 0$
  - $F(\cdot)$ is continuously differentiable
  - $F'(x) > 0$
  - $F''(x) < 0$
  - $\lim_{x \to 0} F'(x) = \infty$
  - $\lim_{x \to \infty} F'(x) = 0$

Capital accumulation

- The stock of capital evolves according to

$$K_{t+1} = I_t + (1 - \delta)K_t$$

  where $\delta \in [0, 1]$ is the rate of depreciation
- Output in the (closed) economy can be used for consumption and saving/investment, so that an additional resource constraint applies:

$$Y_t = C_t + I_t$$

$S_t = I_t$

(CLOSED ECONOMY)

Endowment

- Individuals split their time between working (N_t) and leisure (L_t)

$$N_t + L_t = H$$

  where H is the total amount of hours available, which can be normalized to 1
- Households' utility depends on leisure, so that the time spent to work is endogenous in this economy

Households' preferences

- The economy is populated by a large number of infinitely-lived agents whose expected utility is defined as

$$E_0 \sum_{t=0}^{\infty} b^t U(C_t, L_t) \qquad 0 < b < 1$$

where b is the discount factor, and C_t and L_t represent consumption and leisure at time t

- E_0 denotes expectations based on the information set available at time $t = 0$

Detrendization

A preliminary step before the solution is obtained:

- When the balanced growth restrictions are imposed it is possible to transform the economy by scaling all trending variables $(c_t = C_t/X_t, y_t = Y_t/X_t, k_t = K_t/X_t, i_t = I_t/X_t)$
- Thus, the problem can be rewritten as (*prove it*)

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t), \quad \beta = b\gamma^{1-\sigma}$$

(CRRA)

s.t.

$$\begin{aligned} N_t &= 1 - L_t \\ y_t &= c_t + i_t \\ y_t &= A_t F(k_t, N_t) \\ \gamma k_{t+1} &= i_t + (1-\delta) k_t \end{aligned}$$

(CD)

Solving the model: a competitive equilibrium interpretation

There are several ways of decentralizing the basic RBC model economy. Here we will focus on a sequential competitive equilibrium in which households own both firms and the stock of capital. In this respect, households make three inter-related decisions:

- How much labor to supply (N_t^s)
- How much capital to accumulate (k_{t+1}^s)
- How much to consume (c_t)

Solving the model: a competitive equilibrium interpretation

Households

ENDOGENOUS STATE VARIABLE

- In this decentralization scheme, households have to take into account the law of motion for the wage rate (w_t) and the rental rate (R_t):

$$w_t = w\left(k_t, A_t\right) \qquad R_t = R\left(k_t, A_t\right)$$

EXOGENOUS STATE VARIABLES

- To forecast these prices, agents need to conjecture the functional form of $w\left(\cdot,\cdot\right)$ and $R\left(\cdot,\cdot\right)$, as well as the laws of motion of A_t and k_t
- A_t is assumed exogenous. As such, it evolves independent of k_t

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t$$

AR(1)

- The law of motion for the aggregate capital stock is recursive

$$k_{t+1} = g\left(k_t, A_t\right)$$

Solving the model: a competitive equilibrium interpretation

Households

Households' problem

$$\max_{c_t, N_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$$

s.t.

$$c_t + \gamma k_{t+1} = w_t N_t + (1 + R_t - \delta) k_t + \Pi_t$$

DIVIDENDS

Solving the model: a competitive equilibrium interpretation

Households

- Set up the Lagrangian for this problem

$$\begin{aligned}\mathcal{L}_t &= E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t, L_t) \\ &\quad + \lambda_t \left[w_t N_t + (1 + R_t - \delta) k_t + \Pi_t - c_t - \gamma k_{t+1}\right]\}\end{aligned}$$

- The Lagrange multiplier is $\lambda_t = u'_c(c_t, L_t)$. Therefore:

$$w_t = \frac{u'_L(c_t, L_t)}{u'_c(c_t, L_t)}$$

LS

$$\beta E_t \left[(1 + R_{t+1} - \delta) u'_c(c_{t+1}, L_{t+1})\right] = \gamma u'_c(c_t, L_t)$$

KS + LAW OF MOTION

$$\partial K_{t+1} = i_t + (1-\delta) K_t$$

Solving the model: a competitive equilibrium interpretation

Firms

- Firms' problem reads as:

$$\max_{N_t, k_t} \Pi_t = A_t F\left(k_t, N_t\right) - w_t N_t - R_t k_t$$

- Which returns the familiar conditions:

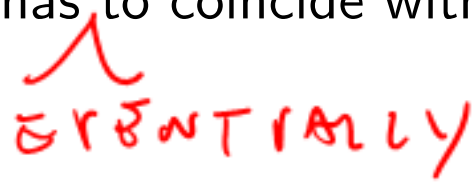
$$\begin{aligned} w_t &= A_t F'_N\left(k_t, N_t\right) \\ R_t &= A_t F'_k\left(k_t, N_t\right) \end{aligned}$$

Market clearing

- We can state the equilibrium conditions by limiting ourselves to the factor markets (due to Walras' law the goods market will also be in equilibrium):

$$N_t^s = N_t^d = N_t, \qquad k_t^s = k_t^d = k_t$$

- To ensure this is a competitive equilibrium under rational expectations, the law of motion conjectured by households (which is required to forecast the future states of the economy, i.e. the future path of prices, $\{w_t, R_t\}_{t=0}^{\infty}$ required for the intertemporal optimization) has to coincide with the actual aggregate law of motion:



$$g(k_t, A_t) = k_{t+1}$$

Welfare

- The economy we have presented satisfies the conditions under which the first welfare theorem applies
- Therefore the competitive equilibrium is the Pareto optimal allocation
- The first-order conditions of the competitive equilibrium coincide with those of the planner's problem

Understanding the dynamics of the RBC model

Intertemporal substitution in labor supply

- Consider the model without uncertainty. By combing the two optimality requirements in the demand side we get

$$\beta(1 + R_{t+1} - \delta) = \frac{\overbrace{u'_c(c_t, L_t)}^{= \frac{u'_L(c_t, L_t)}{w_t}}}{u'_c(c_{t+1}, L_{t+1})}$$

$$\frac{u'_L(c_{t+1}, L_{t+1})}{u'_L(c_t, L_t)} = \frac{1}{\beta(1 + R_{t+1} - \delta)} \frac{w_{t+1}}{w_t}$$

- Under log-utility we get ($u_t = \log c_t + \varphi \log L_t$):

$$\frac{L_t}{L_{t+1}} = \frac{1}{\beta(1 + R_{t+1} - \delta)} \frac{w_{t+1}}{w_t}$$

- Intertemporal substitution in labor supply:
  - If $w_{t+1}/w_t \uparrow \overset{\text{ceteris paribus}}{\Longrightarrow} L_{t+1}/L_t \downarrow$ (substitution effect $>$ income effect)
  - If $R_{t+1} \uparrow \overset{\text{ceteris paribus}}{\Longrightarrow} L_{t+1}/L_t \uparrow$ (compensation effect)

Understanding the dynamics of the RBC model

Intertemporal consumption choices

- Consider separable preferences ($u'_c(c_t, L_t) = u'_c(c_t)$):

$$u'_c(c_t) = \beta E_t[(1 + R_{t+1} - \delta)u'_c(c_{t+1})]$$

- Which can be represented as

$$u'_c(c_t) = \beta E_t[\widetilde{R}_{t+1}]E_t[u'_c(c_{t+1})] + \beta Cov[\widetilde{R}_{t+1}, u'_c(c_{t+1})]$$

  where $\widetilde{R}_{t+1} = 1 + R_{t+1} - \delta$
- Saving attractive when $\widetilde{R}_{t+1}$ highly correlated with $u'_c(c_{t+1})$ (i.e., when future consumption is low). In these circumstances:

$$\text{Savings} \uparrow \Rightarrow c_t \downarrow \Rightarrow u'_c(c_t) \uparrow$$

Calibration

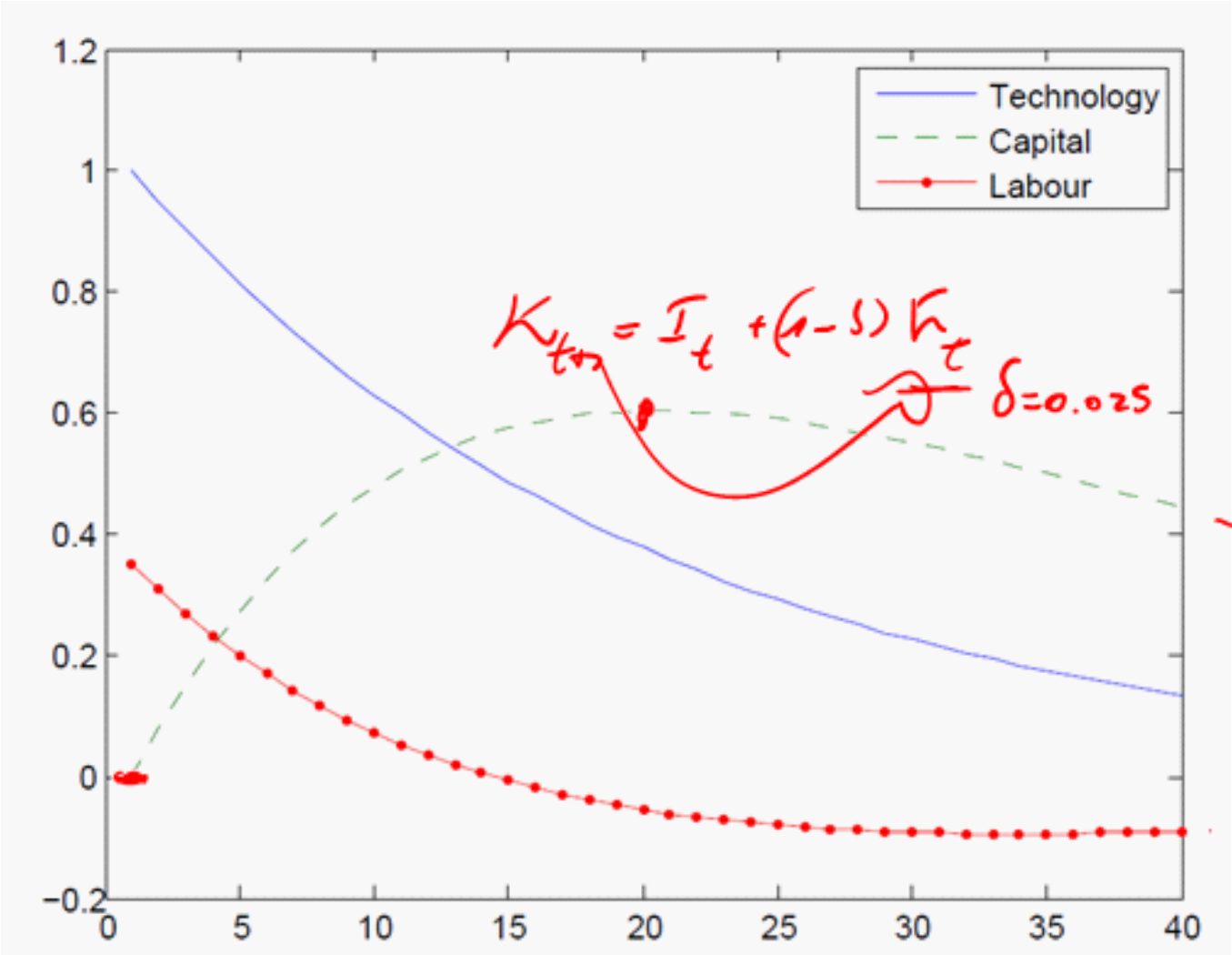
The work of Kydland and Prescott (1982) and Long and Plosser (1983) illustrated the value of exploring stochastic dynamic models by using a "reasonable" set of parameter values. Following the methodological recommendations of Lucas (1980) in his influential "Methods and Problems in Business Cycle Theory," Kydland and Prescott relied on microeconomic empirical studies and the long-run properties of the economy to choose parameter values.

Once we fix the parameters value we can:

1. Investigate the transmission mechanism of the shocks
2. Check whether the model displays the same statistical properties of the data (match the basic moments: relative variances, correlations and autocorrelations....)
3. If we include the entire path of the shocks we can see whether the model replicates actual fluctuations in the economy

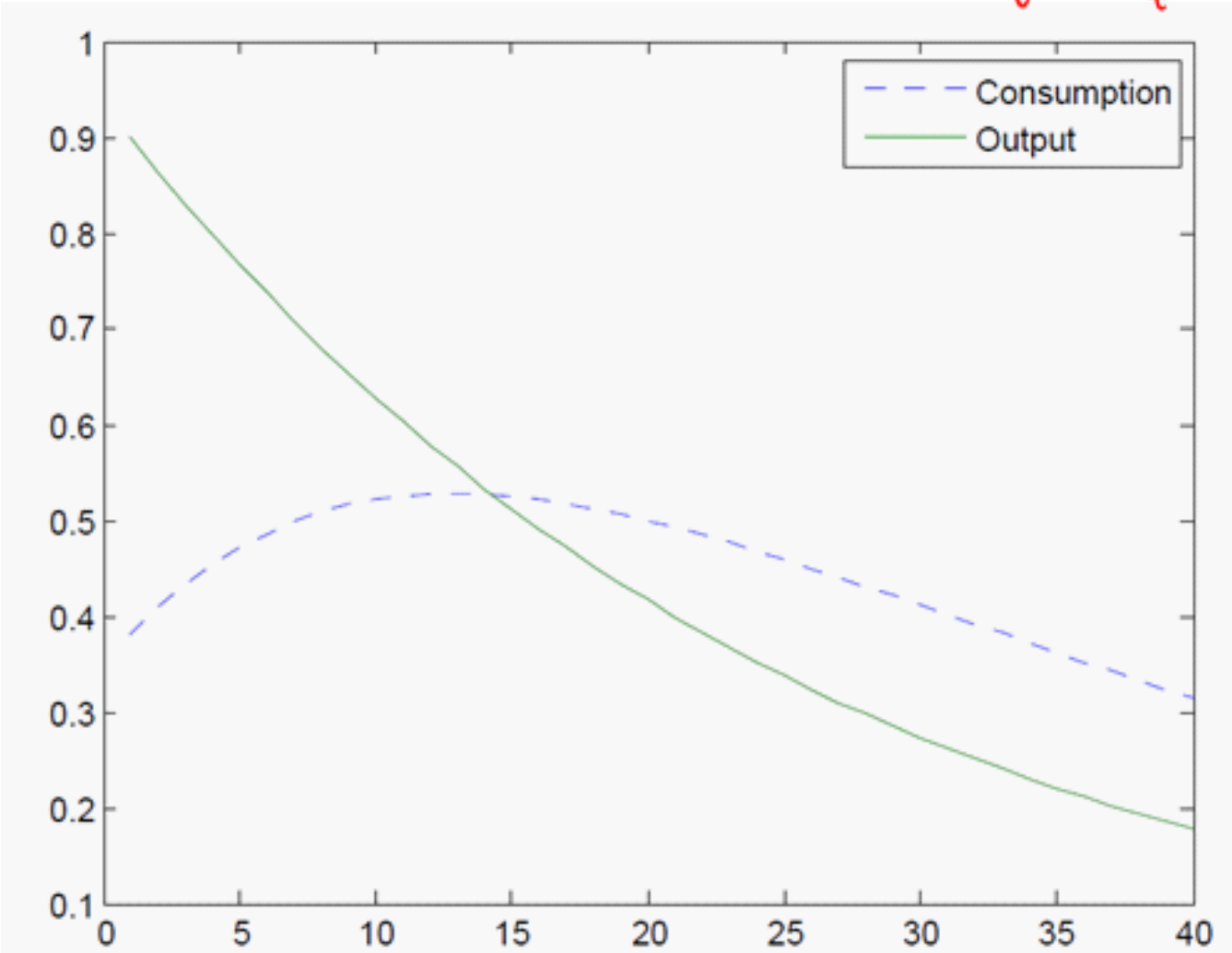
The effect of a technology shock

$\log A_t = \rho \log A_{t-1} + \varepsilon_t$



The effect of a technology shock

$Y_t = C_t + I_t$



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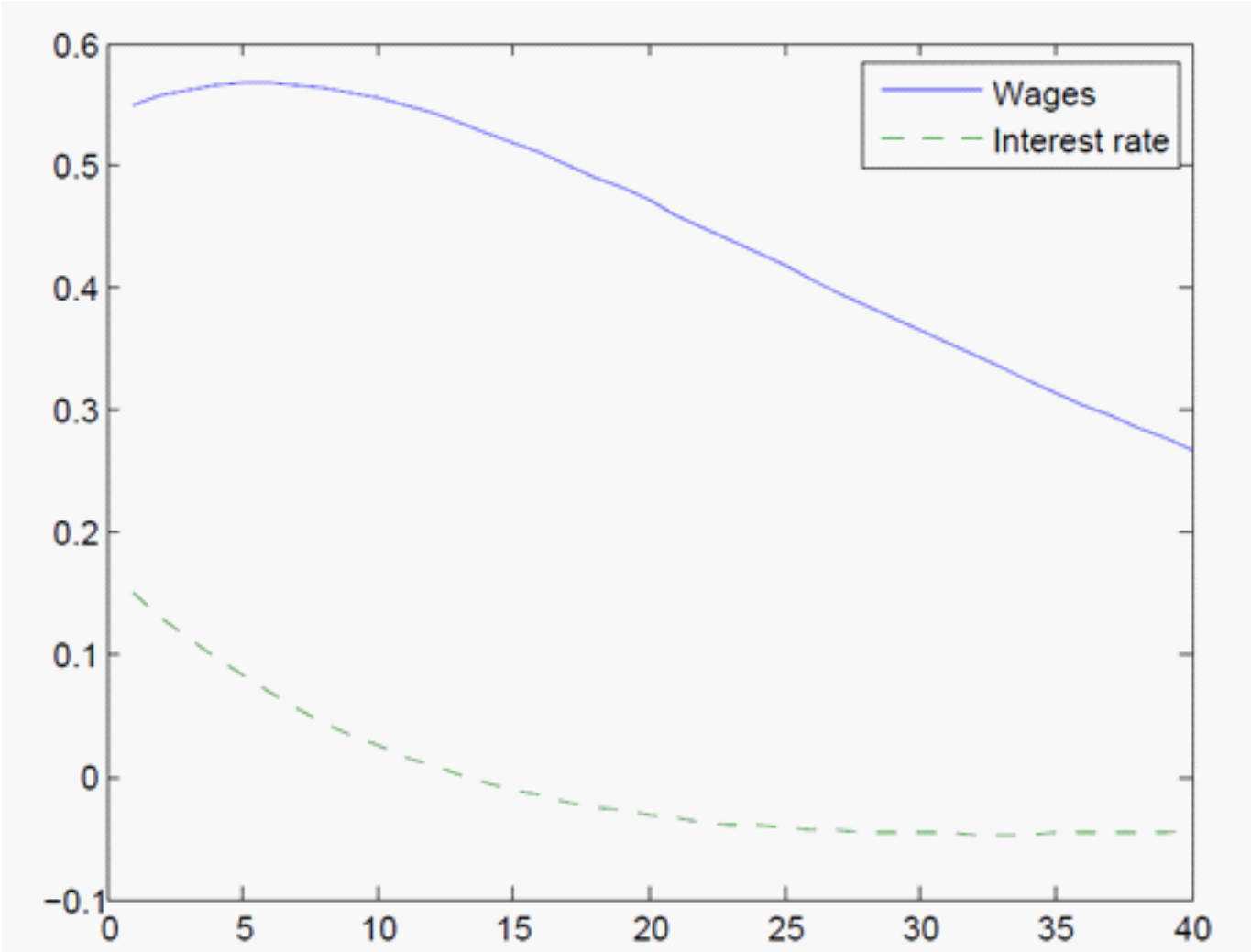

Does the model generate business cycles?

Surprisingly Yes!! (but....)

TABLE 4.4 A calibrated real-business-cycle model versus actual data

	U.S. data	Baseline real-business-cycle model
σ_Y	1.92	1.30
σ_C/σ_Y	0.45	0.31
σ_I/σ_Y	2.78	3.15
σ_L/σ_Y	0.96	0.49
Corr(L, Y/L)	−0.14	0.93

Source: Hansen and Wright (1992).

$L \equiv$ LABOR HS.

Main implications

Prescott (1986, 13)

- *"Economic theory implies that, given the nature of the shocks to technology and people willingness and ability to intertemporally and intratemporally substitute, the economy will display fluctuations like those the U.S. economy displays."*

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- *"...theory predicts what is observed. Indeed, if the economy did not display the business cycle phenomena, there would be a puzzle."*
- *"Economic fluctuations are optimal responses to uncertainty in the rate of technological change."*

Policy implications

Prescott (1986)

"The policy implication of this research is that costly efforts at stabilization are likely to be counterproductive."

Criticisms

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Criticisms

1. The model performance requires an empirically unreasonable degree of intertemporal substitution in labor supply
2. Counterfactual implications for some relative and absolute prices:
   1- Strongly procyclical real wage
   2 -Euler equation is incompatible with the Equity Premium Puzzle
3. Technology shocks are the dominant source of fluctuations and they must be very large and persistent

Extensions to the baseline Neoclassical (RBC) model

Employment variability

- Data: employment is as much volatile as output and is strongly procyclical, whereas real wage is less volatile and only weakly procyclical
- RBC: the observed pattern is obtained only by assuming a very large wage elasticity of labor supply (which is not supported by the microeconomic evidence)
- However: by introducing indivisibility (non convexity) in labor supply decisions (i.e. workers can choose whether to work or not to work, but not how many hours to work per week), it is possible to reconcile a highly volatile employment with a low labor supply elasticity

Extensions to the baseline Neoclassical (RBC) model

Productivity

- Data: labor productivity and employment are not highly correlated
- RBC: large correlation between productivity and employment (due to technology shocks)
- However: productivity-employment correlation can be reduced by the introduction of shocks to labor supply (*ad hoc*?)

Extensions to the baseline Neoclassical (RBC) model

Capacity utilization

- The short-run elasticity of capital supply is zero: there is no way for the economy to increase the capital stock inherited from the previous period
- The long-run elasticity of capital supply is infinity
- The evidence suggests that the flow of capital services is high in expansions. In contrast, recessions are times when capital tends to lie idle, thus producing a small service flow

Extensions to the baseline Neoclassical (RBC) model

Capacity utilization (2)

- Production function with variable capital utilization

$$y_t = A_t F\left(U_t k_t, N_t\right)$$

- The low of motion of capital modified to adjust for state dependent depreciation:

$$\gamma k_{t+1} = i_t + \left[1 - \delta\left(U_t\right)\right] k_t$$

where $\delta\left(U_t\right)$ is a convex and increasing function of utilization

Reading list

- Main reference: Romer, ch. 5.3-5.5 (though in continuous time, this is the most palatable version)
- Further reading: King, Plosser and Rebelo (KPR1988), Sections 1-2