Macroeconomics III

Lecture 8

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Outline

- A primer on rational expectations and expectational difference equations (BF 5.1 on fundamental solutions; *exclude* bubbles, multivariate models and indeterminate solutions)
- Lucas model of imperfect information (DR 6.9-6.10)
- Another source of nominal rigidity: Fisher contracts (DR 7.1-7.2)

Rational expectations: a primer

- When there is uncertainty we do not know future outcomes but must make *expectations* about them
- We say these expectations are *rational* when we assume they are formed by:
  - Knowing the structure of the underlying economy (i.e., the model)
  - Using all the available information
- Under RE people do not make systematic forecast errors (unpredictability of forecast errors)

Rational expectations

- Under uncertainty economic variables behave as *stochastic variables* and RE of a given variable X at $t+1$ are represented as the mathematical expectation
$$E_t[X_{t+1}] = E[X_{t+1}|I_t]$$
where I_t represents information known up to time t
- Examples:
  - Efficient market hypothesis $\rightarrow$ q_t reflects information at time t on future dividends
  - Life-cycle permanent income hypothesis $\rightarrow$ C_t random walk process, so that changes in consumption are unpredictable

Expectational Difference Equations (EDE)

- EDE feature conditional expectations of endogenous variables, y, that we want to solve for to get their distribution as a function of the distribution of exogenous ones, x

$$y_t = aE_t[y_{t+1}] + cx_t \tag{1}$$

- Assuming RE means that $E_t[y_{t+1}]$ is the mathematical expectation of y_{t+1} given I_t, and knowing the underlying economic model, i.e. knowing equation (1)
- The information set includes all past values of y_t and x_t

Expectational difference equations

- To solve (1) we start using RE to estimate $E_t[y_{t+1}]$. Since agents know the model, they take relation (1) shifted one period ahead as satisfied by y_{t+1} (solve, replace y_{t+1} using (1) for period $t+1$, then take expectations conditional on information in period t)
- Thus:

$$y_t = a^2 E_t[y_{t+2}] + acE_t[x_{t+1}] + cx_t$$

Expectational difference equations

- Repeating the procedure (repeat T times) and assuming $\lim_{T\to\infty} a^T E_t[y_{t+T}] = 0$, we arrive at the desired solution

$$y_t = c \sum_{i=0}^{\infty} a^i E_t[x_{t+i}]$$

  where the variable y_t depends on current and future expected values of x
- Application: asset prices and dividends

Expectational difference equations

- Assume a stochastic process for x_t:

$$x_t - x = \rho\,(x_{t-1} - x) + u_t\,, \quad \text{with} \quad E[\,u_t |\, I_{t-1}] = 0$$

- Using the law of iterated expectations:

$$E[\,x_{t+i} |\, I_t] = x + \rho^i\,(x_t - x)$$

- Assume $a\rho < 1$:

$$\begin{aligned}
y_t &= c\sum_{i=0}^{\infty} a^i E_t[x_{t+i}] \\
&= c\sum_{i=0}^{\infty} a^i\left[x + \rho^i\,(x_t - x)\right] = \underbrace{c\sum_{i=0}^{\infty} a^i x}_{=\frac{cx}{1-a}=y} + \underbrace{c\sum_{i=0}^{\infty} a^i \rho^i\,(x_t - x)}_{=c\frac{x_t - x}{1-a\rho}} \\
&= y + \frac{c}{1-a\rho}\,(x_t - x)
\end{aligned}$$

Next application: Lucas model

- Lucas (1972) constructed this model to obtain a Phillips curve in an equilibrium with RE
- Nevertheless, even if there is a positive relation between inflation and output, a policy maker with no informational advantage cannot use it to engineer a boom (very relevant for the analysis in the last part of the course)
- Agents do not suffer from “money illusion"

Key modeling aspects

- Many competitive producers that are unsure about whether changes in their own price reflect a change in the demand for their own product (which depends on the relative price) or changes in the aggregate price level
- Ideally, production should be adjusted with respect to movements in relative prices, but not with respect to changes in the general price level
- In reality (i.e., under uncertainty), an increase in the observed price implies that, with some probability, demand is higher, and thus production will be increased proportionally to this probability
- The Lucas model solves this decision problem and delivers some crucial macroeconomic implications

Relative price and price level

- When the price of a given producer's good increases, there is some chance that the increase reflects a rise in the **price level**, and some chance that it reflects a rise in the good's **relative price**
- The rational response for the producer is to attribute part of the change to an increase in the price level and part to an increase in the relative price, and therefore to increase output somewhat
- As a result, an increase in aggregate demand that is not publicly observed leads to some combination of a rise in the overall price level and a rise in overall output

Baseline structure

- Each household is producing and consuming at the same time, maximizing

$$\max_{C_i, L_i} U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$$

where C is consumption of a basket of goods and L is labor supply

- Price taking
- Linear production technology: $Y_i = L_i$
- Assume also that the price of the basket of goods being consumed is equal to the avegare price of all goods

Optimal decision rules

- Plug total (real) revenues ($C_i = \frac{P_i}{P} Y_i$) and the production techology into the utility function

$$U_i = \frac{P_i}{P} Y_i - \frac{1}{\gamma} Y_i^{\gamma}$$

- FOC:

$$\frac{P_i}{P} = Y_i^{\gamma - 1}$$

Linearization

- Rearranging and taking logs (variables in lowercase are logs of uppercase variables), optimal production is given by:

$$y_i = \frac{1}{\gamma - 1}(p_i - p) \tag{2}$$

- Demand for goods in market i is given by

$$y_i = y + z_i - \eta(p_i - p) = m - p + z_i - \eta(p_i - p) \tag{3}$$

  where $y = m - p$ (quantity theory of money), z_i is a taste shock, and $\eta > 0$

- Producer i cannot observe z_i and m, but observes $p_i = p + (p_i - p)$ and must infer the size of change in the relative price $r_i \equiv p_i - p$ which is what determines optimal production, see (2)

Solving the model

- With no uncertainty the production level would simply depend on r_i
- However, the producer needs to base his/her decisions on $E[r_i|p_i]$
- We assume that, after $E[r_i|p_i]$ is estimated, the producer takes this expectation for granted (this certainty-equivalence setting is not identical to maximizing expected utility, but represents a reasonable approximation)

Solving the model

- To determine $E[r_i|p_i]$ we postulate that m and z_i are independent random variables with normal distributions

$$\begin{aligned} m &\sim N(E(m), V_m) \\ z_i &\sim N(0, V_z) \end{aligned}$$

- We then assume that p and r_i are independent and normally distributed variables (we will later verify the conjecture), and use the following property

$$\text{If} \quad \begin{pmatrix} x \\ y \end{pmatrix} \sim N\left(\begin{pmatrix} E[x] \\ E[y] \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right)$$
$$\rightarrow \quad E[x|y] = E[x] + \frac{\Sigma_{11}}{\Sigma_{22}}(y - E[y])$$

Solving the model

- In our case, where p_i equals r_i plus an independent variable, p:

$$E[r_i|p_i] = E[r_i] + \frac{V_r}{V_r + V_p}(p_i - E[p])$$

  Why?

- *Under the assumption of certainty equivalence* we replace r_i by $E[r_i|p_i]$ in (2) (again: this is an "approximation", we should maximize $E[U_i|P_i]$)

$$y_i = \frac{1}{\gamma - 1}\frac{V_r}{V_r + V_p}(p_i - E[p]) \equiv b(p_i - E[p]) \qquad (4)$$

- Aggregating over all producers, $p = \bar{p}_i$, we get the "Lucas supply curve"

$$y = b(p - E[p]) \qquad (5)$$

- As we will see, this equation provides a microfoundation for the Phillips curve

General equilibrium

- Supply equals demand

$$b(p - E[p]) = m - p$$

- Solution:

$$\begin{aligned} p &= \frac{1}{1+b}m + \frac{b}{1+b}E[p] \\ y &= \frac{b}{1+b}m - \frac{b}{1+b}E[p] \end{aligned}$$

General equilibrium

- Take $p = \frac{1}{1+b}m + \frac{b}{1+b}E[p]$ to find $E[p]$:

$$E[p] = \frac{1}{1+b}E\left[m\right] + \frac{b}{1+b}E[p] \tag{6}$$

- Thus

$$E\left[p\right] = E\left[m\right]$$

General equilibrium

- Using the fact that $m = E[m] + (m - E[m])$:

$$p = E[m] + \frac{1}{1+b}(m - E[m]) \quad (7)$$
$$y = \frac{b}{1+b}(m - E[m])$$

- Thus *expected* money demand, $E(m)$, only affects prices, while its *unexpected* component affects both p and y
- Note that (7) confirms us that p is normally distributed, as initially assumed

General equilibrium: implications (1)

- Assume an unexpected increase in m:

$$p = \underset{=}{E[m]} + \frac{1}{1+b}(\underset{\uparrow}{m} - \underset{=}{E[m]}) \tag{8}$$

$$y = \frac{b}{1+b}(\underset{\uparrow}{m} - \underset{=}{E[m]}) \tag{9}$$

- The increase in the money supply raises aggregate demand, and thus produces an outward shift in the demand curve for each good
- Since the increase is not observed, each supplier's best guess is that some portion of the rise in the demand for his or her product reflects a relative price shock
- Thus, producers increase their output

General equilibrium: implications (2)

- Assume an observed increase in m, with $m - E[m]$ held fixed:

$$p = \underset{\uparrow}{E[m]} + \frac{1}{1+b}(m - \underset{=}{E}[m]) \tag{10}$$

$$y = \frac{b}{1+b}(m - \underset{=}{E}[m]) \tag{11}$$

- In this case, each supplier attributes the rise in the demand for his or her product to money growth, and thus does not change his or her output
- Thus, observed changes in aggregate demand only affect prices

Structural representation

- To complete the solution of the model, we need to find b as a function of the fundamentals, V_z and V_m
- To solve for r_i, we take (3) and (4):

$$
\begin{aligned}
y_i &= m - p + z_i - \eta(p_i - p) \\
y_i &= b(p_i - p) + b(p - E[p])
\end{aligned}
$$

- Combine them to eliminate y_i :

$$
\begin{aligned}
br_i + b(p - E[p]) &= m - p + z_i - \eta r_i \\
(b + \eta)\, r_i &= \underbrace{-b(p - E[p]) + m - p}_{=0} + z_i \\
r_i &= \frac{z_i}{\eta + b}
\end{aligned}
$$

- Thus, $V_r = \frac{V_z}{(\eta+b)^2}$
- Note that r_i is normally distributed and independent of p

Structural representation

- Finally, from (7) ($p = E[m] + \frac{1}{1+b}(m - E[m])$) and recalling that $V_x = E[x^2] - E^2[x]$:

$$V_p = \frac{V_m}{(1+b)^2}$$

  you can show this...

- Thus, we have found V_p and V_r as functions of V_z and V_m

Business cycles

- Replacing in equation (4)

$$b = \frac{1}{\gamma - 1}\left[\frac{V_z}{V_z + \frac{(\eta+b)^2}{(1+b)^2}V_m}\right]$$

which implicitly gives b as a function of V_z and V_m

- It can be shown that

$$\frac{db}{dV_z} > 0, \quad \frac{db}{dV_m} < 0$$

Phillips curve

- We have uncovered a positive relation between output and innovations in the price level
- A proper Phillips curve is one step away, and requires a specification for aggregate demand
- We take money to behave as a random walk with drift (u_t is a white noise)

$$m_t = m_{t-1} + c + u_t$$

- Thus $E[m_t] = m_{t-1} + c$, and the equilibrium equations become:

$$p_t = E[m_t] + \frac{1}{1+b}(m_t - E[m_t]) = m_{t-1} + c + \frac{1}{1+b} u_t \quad (12)$$

$$y_t = \frac{b}{1+b}(m_t - E[m_t]) = \frac{b}{1+b} u_t \quad (13)$$

Phillips curve

- Lagging and subtracting (12) from itself we get inflation $\pi_t \equiv p_t - p_{t-1}$ (solve)

$$\begin{aligned} \pi_t &= m_{t-1} - m_{t-2} + \frac{1}{1+b}\left(u_t - u_{t-1}\right) \\ &= c + \frac{b}{1+b} u_{t-1} + \frac{1}{1+b} u_t \end{aligned}$$

- Two more steps:
  - Recall (13) and note that $\frac{1}{1+b} u_t = \frac{1}{b} y_t$
  - From the solution of inflation, $E_{t-1}[\pi_t] = c + \frac{b}{1+b} u_{t-1}$
- Thus, we see that output and inflation are positively correlated $\longrightarrow$ Phillips curve:

$$\pi_t = E_{t-1}[\pi_t] + \frac{1}{b} y_t$$

Phillips curve and policy stabilization trade-off

- Although there is a positive relationship between π and y, only unobserved aggregate demand shocks have real effects
- Suppose average money growth is raised (i.e., $c\uparrow$):
  - If the change is unknown to the public: unobserved money growth is temporarily higher and output is above normal
  - If the change is known to the public: expected money growth jumps immediately and no temporary boom is experienced

A policy change unknown to the public

- Assume there is an unforeseen monetary injection at time t (i.e., $c_t > c$)
- Equilibrium conditions at $t-1$:

$$
\begin{aligned}
p_{t-1} &= m_{t-2} + c + \frac{1}{1+b} u_{t-1} \\
y_{t-1} &= \frac{b}{1+b} u_{t-1}
\end{aligned}
$$

- Equilibrium conditions at t:

$$
\begin{aligned}
p_t &= m_{t-1} + c_t + \frac{1}{1+b} u_t \\
y_t &= \frac{b}{1+b} u_t
\end{aligned}
$$

A policy change unknown to the public (contd.)

- Inflation:

$$\pi_t = \underbrace{m_{t-1}}_{=m_{t-2}+c+u_{t-1}} - m_{t-2} + \frac{1}{1+b}\left(u_t - u_{t-1}\right) + c_t - c$$

- Thus:

$$\pi_t = c_t + \frac{b}{1+b}u_{t-1} + \frac{1}{1+b}u_t$$

- Once again, $\frac{1}{1+b}u_t$ becomes $\frac{1}{b}y_t$ after multiplying and dividing by b:

$$\pi_t = c_t + \frac{b}{1+b}u_{t-1} + \frac{1}{b}y_t$$

- Now, add and subtract c and note that $E_{t-1}[\pi_t] = c + \frac{b}{1+b}u_{t-1}$ in the event of an unforeseen policy change:

$$\pi_t = c_t - c + E_{t-1}\left[\pi_t\right] + \frac{1}{b}y_t$$

A policy change known to the public

- Assume there is a foreseen monetary injection
- Equilibrium conditions at t:

$$\begin{aligned} p_t &= m_{t-1} + c_t + \frac{1}{1+b} u_t \\ y_t &= \frac{b}{1+b} u_t \end{aligned}$$

- Inflation:

$$\pi_t = \underbrace{m_{t-1}}_{= m_{t-2} + c + u_{t-1}} - m_{t-2} + \frac{1}{1+b}\left(u_t - u_{t-1}\right) + c_t - c$$

- Thus:

$$\pi_t = c_t + \frac{b}{1+b} u_{t-1} + \frac{1}{1+b} u_t$$

- Once again, $\frac{1}{1+b} u_t$ becomes $\frac{1}{b} y_t$ after multiplying and dividing by b:

$$\pi_t = c_t + \frac{b}{1+b} u_{t-1} + \frac{1}{b} y_t$$

- Now, as the policy change is foreseen $E_{t-1}\left[\pi_t\right] = c_t + \frac{b}{1+b} u_{t-1}$ and:

$$\pi_t = E_{t-1}\left[\pi_t\right] + \frac{1}{b} y_t$$

Phillips curve and policy stabilization

- Monetary policy can stabilize/stimulate real activity only if policy-makers have information that is not available to private agents
- The basic idea is more general. When expectations influence equilibrium, changes in policy will affect expectations and thus the statistical relations between economic outcomes break down
- This is the *Lucas critique* (1976) that tells us not to mechanically extrapolate past behavior into the future

Empirical prediction

- The Lucas (1972) model predicts that in economies with high aggregate demand volatility (high V_m) the real effects of a given change in aggregate demand should be smaller (recall $\partial b / \partial V_m < 0$)
- Lucas (1973) tests this prediction using cross-country data
- Although there is some positive evidence, later studies show that nominal rigidities in price setting have more explanatory power
- Perhaps we should move away from competitive behavior and assume firms have *market power in setting prices*

Price setting

- For a fully fledged dynamic model, see DR 7.1 (dynamic version of the one examined in Lecture 8). Today, we just give a primer
- The underlying structure is similar to the Lucas model (households derive utility from consumption of a basket of goods, and do not like to work)

Modeling price setting

- The representative agent i maximizes utility

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$$

  subject to the constraint

$$C_i = \frac{P_i}{P} Y_i$$

  where C_i is consumption, L_i labor supply, P the aggregate price level, P_i the price of good i and Y_i the quantity of good i. The production function equals

$$Y_i = L_i$$

- We have monopolistic competition in the goods market. *Additional constraint*: demand for good i is (ignore idiosyncratic shocks)

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$$

Modeling price setting

- Substitute the budget constraint, the technology constraint and the demand function into the utility function, so as to get:

$$U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\gamma} Y_i^{\gamma}$$

- Maximization w.r.t. Y_i:

$$\frac{\partial U_i}{\partial Y_i} = 0 \Rightarrow -\frac{1}{\eta}\left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}-1} Y_i + \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}} - Y_i^{\gamma-1} = 0$$

- Rearrange:

$$\begin{aligned}
(Y_i)^{\gamma-1} &= \left(1 - \frac{1}{\eta}\right)\left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}} \\
(Y_i)^{\gamma-1} &= \left(1 - \frac{1}{\eta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} \\
(Y_i)^{\gamma-1} &= \left(1 - \frac{1}{\eta}\right)\frac{P_i}{P} \\
Y_i &= \left(1 - \frac{1}{\eta}\right)^{\frac{1}{\gamma-1}}\left(\frac{P_i}{P}\right)^{\frac{1}{\gamma-1}}
\end{aligned}$$

Modeling price setting

- Desired price at the individual level:

$$p_i^* - p = (\gamma - 1)\, y_i \underbrace{- \ln\left(1 - \frac{1}{\eta}\right)}_{\equiv \mu}$$

- All households/firms charge the same amount and produce the same amount:

$$p_i^* - p = (\gamma - 1) \underbrace{y}_{=m-p} + \mu$$

- Denoting $\phi = \gamma - 1$:

$$p^* = \phi m + (1 - \phi)\, p \tag{14}$$

  where we have ignored the constant, and $\phi \geq 0$ measures the degree of real rigidity (inverse relationship)
- Why? Example: Higher demand induces higher production, and since the marginal disutility from labor increases in L_i, a higher wage rate is required to obtain more labor hours. These higher costs pass into a higher price for the ith good, for ϕ relatively high. For ϕ relatively low, instead, prices display lower reactiveness to changes in aggregate demand

Modeling price setting

- To study the effects of demand shocks we postulate that m is random (need not to impose a Normal distribution)
- If price-setters can choose p_i every period, they must *form expectations on m and on how other price-setters behave*
- So (14) gives desired prices, p_i^*, and actual prices set are $p_i = E[p_i^*|I]$

$$p_i = \phi E[m|I] + (1-\phi)E[p|I]$$

- Assume everybody behaves in the same way, so that $p_i = p$. Thus, taking expectations

$$E[p|I] = E[m|I]$$

Modeling price setting

- So, the equilibrium is

$$
\begin{aligned}
p &= E[m|I] \\
y &= m - E[m|I]
\end{aligned}
$$

- Equilibrium has the same crucial property as the Lucas model: only unanticipated shocks to aggregate demand have real effects
- Market power does not alter the baseline insight. What's next then?
- For anticipated shocks to have real effects we need to introduce *frictions* in price setting, so not all firms set prices each period
- For simplicity we assume that prices are set by some time dependent rule, not as a response to economic conditions