

Macroeconomics III

Lecture 9

Emiliano Santoro

University of Copenhagen

Fall 2021

Outline

- Recap
- Another source of nominal rigidity: Fisher contracts and policy stabilization (DR 7.1-7.2)
- Fixed contracts (DR 7.3)
- A step towards a proper dynamic model with nominal rigidities: Calvo price-setting and the New Keynesian Phillips curve (DR 7.4)

Phillips curve and policy stabilization

- Monetary policy can stabilize/stimulate real activity only if policy-makers have information that is not available to private agents
- The basic idea is more general. When expectations influence equilibrium, changes in policy will affect expectations and thus the statistical relations between economic outcomes break down
- This is the *Lucas critique* (1976) that tells us not to mechanically extrapolate past behavior into the future

Empirical prediction

- The Lucas (1972) model predicts that in economies with high aggregate demand volatility (high V_m) the real effects of a given change in aggregate demand should be smaller (recall $\partial b / \partial V_m < 0$)
- Lucas (1973) tests this prediction using cross-country data
- Although there is some positive evidence, later studies show that nominal rigidities in price setting have more explanatory power
- Perhaps we should move away from competitive behavior and assume firms have *market power in setting prices*

Price setting

- For a fully fledged dynamic model, see DR 7.1 (dynamic version of the one examined in Lecture 8). Today, we just give a primer
- The underlying structure is similar to the Lucas model (households derive utility from consumption of a basket of goods, and do not like to work)

Modeling price setting

- The representative agent i maximizes utility

$$U_i = C_i - \frac{1}{\gamma} L_i^\gamma$$

subject to the constraint

$$C_i = \frac{P_i}{P} Y_i$$

where C_i is consumption, L_i labor supply, P the aggregate price level, P_i the price of good i and Y_i the quantity of good i . The production function equals

$$Y_i = L_i$$

- We have monopolistic competition in the goods market. *Additional constraint*: demand for good i is (ignore idiosyncratic shocks)

$$Y_i = \left(\frac{P_i}{P} \right)^{-\eta} Y$$

Modeling price setting

- Substitute the budget constraint, the technology constraint and the demand function into the utility function, so as to get:

$$U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\gamma} Y_i^\gamma$$

- Maximization w.r.t. Y_i :

$$\frac{\partial U_i}{\partial Y_i} = 0 \Rightarrow -\frac{1}{\eta} \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}-1} Y_i + \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}} - Y_i^{\gamma-1} = 0$$

- Rearrange:

$$(Y_i)^{\gamma-1} = \left(1 - \frac{1}{\eta}\right) \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}}$$

$$(Y_i)^{\gamma-1} = \left(1 - \frac{1}{\eta}\right) \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}}$$

$$(Y_i)^{\gamma-1} = \left(1 - \frac{1}{\eta}\right) \frac{P_i}{P}$$

$$Y_i = \left(1 - \frac{1}{\eta}\right)^{\frac{1}{\gamma-1}} \left(\frac{P_i}{P}\right)^{\frac{1}{\gamma-1}}$$

Modeling price setting

- Desired price at the individual level:

$$p_i^* - p = (\gamma - 1) \underbrace{y_i}_{\equiv \mu} - \ln \left(1 - \frac{1}{\eta} \right)$$

- All households/firms charge the same amount and produce the same amount:

$$p_i^* - p = (\gamma - 1) \underbrace{y}_{=m-p} + \mu$$

- Denoting $\phi = \gamma - 1$:

$$p^* = \phi m + (1 - \phi) p \tag{1}$$

where we have ignored the constant, and $\phi \geq 0$ measures the degree of real rigidity (inverse relationship)

- Why?** Example: Higher demand induces higher production, and since the marginal disutility from labor increases in L_i , a higher wage rate is required to obtain more labor hours. These higher costs pass into a higher price for the i^{th} good, for ϕ relatively high. For ϕ relatively low, instead, prices display lower reactivity to changes in aggregate demand

Modeling price setting

- To study the effects of demand shocks we postulate that m is random (need not to impose a Normal distribution)
- If price-setters can choose p_i every period, they must *form expectations on m and on how other price-setters behave*
- So (1) gives desired prices, p_i^* , and actual prices set are $p_i = E[p_i^*|I]$

$$p_i = \phi E[m|I] + (1 - \phi)E[p|I]$$

- Assume everybody behaves in the same way, so that $p_i = p$. Thus, taking expectations

$$E[p|I] = E[m|I]$$

Modeling price setting

- So, the equilibrium is

$$p = E[m|I]$$

$$y = m - E[m|I]$$

- Equilibrium has the same crucial property as the Lucas model: only unanticipated shocks to aggregate demand have real effects
- Market power does not alter the baseline insight. What's next then?
- For anticipated shocks to have real effects we need to introduce *frictions* in price setting, so not all firms set prices each period
- For simplicity we assume that prices are set by some time dependent rule, not as a response to economic conditions

Predetermined prices: the Fischer model

- In the Fischer model each price-setter sets prices for two periods, being able to set different prices for these periods
- For symmetry we assume $\frac{1}{2}$ of producers set prices in odd periods, the other half in even ones
- We assume rational expectations in price setting, i.e. *prices are set using all available information and knowing how other price setters behave*
- Again, (1) should be read as giving desired prices, while actual prices are conditional on the information available

Predetermined prices: the Fischer model

- Let's call p_t^i prices set for period t with information available at time $t - i$
- We thus have the following structure for information and price setting

$t - 1$	t	$t + 1$
l_{t-1}	l_t	l_{t+1}
	p_t^1	p_{t+1}^1
	p_t^2	p_{t+1}^2

Predetermined prices: the Fischer model

- So

$$p_t = \frac{1}{2}(p_t^1 + p_t^2)$$

$$p_t^* = \phi m_t + (1 - \phi) \frac{1}{2}(p_t^1 + p_t^2)$$

- and

$$p_t^1 = E_{t-1}[p_t^*] = \phi E_{t-1}[m_t] + (1 - \phi) \frac{1}{2}(p_t^1 + p_t^2) \quad (2)$$

$$p_t^2 = E_{t-2}[p_t^*] = \phi E_{t-2}[m_t] + (1 - \phi) \frac{1}{2}(E_{t-2}[p_t^1] + p_t^2) \quad (3)$$

Predetermined prices: the Fischer model

- Rearrange both equations:

$$p_t^1 = \frac{2\phi}{1+\phi} E_{t-1}[m_t] + \frac{1-\phi}{1+\phi} p_t^2$$

$$p_t^2 = \frac{2\phi}{1+\phi} E_{t-2}[m_t] + \frac{1-\phi}{1+\phi} E_{t-2}[p_t^1]$$

- Now, find $E_{t-2}p_t^1$, recalling that $E_{t-2}E_{t-1}m_t = E_{t-2}m_t$:

$$E_{t-2}p_t^1 = \frac{2\phi}{1+\phi} E_{t-2}[m_t] + \frac{1-\phi}{1+\phi} p_t^2$$

- Take this and plug it into p_t^2 :

$$p_t^2 = E_{t-2}m_t$$

- Thus

$$p_t^1 = E_{t-2}[m_t] + \frac{2\phi}{1+\phi} (E_{t-1}[m_t] - E_{t-2}[m_t])$$

Predetermined prices: the Fischer model

- Finally, equilibrium price level and output are

$$p_t = E_{t-2}[m_t] + \frac{\phi}{1+\phi} (E_{t-1}[m_t] - E_{t-2}[m_t])$$
$$y_t = m_t - E_{t-1}[m_t] + \frac{1}{1+\phi} (E_{t-1}[m_t] - E_{t-2}[m_t])$$

- So **unanticipated demand shocks** have real effects, as before
- But now also **anticipated shocks** have real effects (information about m_t that becomes available between $t - 2$ and $t - 1$).
- **Why?** Prices are not fully flexible in the short run

Predetermined prices: the Fischer model

- Why a fraction $\frac{\phi}{1+\phi}$ of new information is passed into prices and $\frac{1}{1+\phi}$ into output?
- Because ϕ is an inverse function of the degree of real rigidity, thus accounting for the responsiveness of individual prices to aggregate demand
- If prices are more responsive (i.e., a relatively high ϕ) then there is less of an effect on output, and viceversa

Fischer model with demand shocks

- We postulate the following relationship for aggregate demand

$$y_t = m_t - p_t + v_t$$

where now m_t represents policy effects on aggregate demand (e.g. through changes in money supply) and v_t represents shocks on aggregate demand unrelated to policy

- Aggregate price log-level

$$p_t = \frac{1}{2}(p_t^1 + p_t^2)$$

- As $p_t^* - p_t = \phi y_t$ and $y_t = m_t - p_t + v_t$:

$$p_t^* = \phi(m_t + v_t) + (1 - \phi)\frac{1}{2}(p_t^1 + p_t^2)$$

and

$$p_t^1 = E_{t-1}[p_t^*] = \phi E_{t-1}[m_t + v_t] + (1 - \phi)\frac{1}{2}(p_t^1 + p_t^2) \quad (4)$$

$$p_t^2 = E_{t-2}[p_t^*] = \phi E_{t-2}[m_t + v_t] + (1 - \phi)\frac{1}{2}(E_{t-2}[p_t^1] + p_t^2) \quad (5)$$

Fischer model with demand shocks

- Solving first for (4), this can be plugged in (5).

$$p_t^1 = \frac{2\phi}{1+\phi} E_{t-1}[m_t + v_t] + \frac{1-\phi}{1+\phi} p_t^2$$

$$p_t^2 = \frac{2\phi}{1+\phi} E_{t-2}[m_t + v_t] + \frac{1-\phi}{1+\phi} E_{t-2}[p_t^1]$$

- Now, find $E_{t-2}[p_t^1]$, recalling that $E_{t-2}E_{t-1}[m_t + v_t] = E_{t-2}[m_t + v_t]$:

$$E_{t-2}[p_t^1] = \frac{2\phi}{1+\phi} E_{t-2}[m_t + v_t] + \frac{1-\phi}{1+\phi} p_t^2$$

- Take this equation and plug it into p_t^2 :

$$p_t^2 = E_{t-2}[m_t + v_t]$$

- Thus

$$p_t^1 = E_{t-2}[m_t + v_t] + \frac{2\phi}{1+\phi} (E_{t-1}[m_t + v_t] - E_{t-2}[m_t + v_t])$$

Fischer model with demand shocks

Finally, the equilibrium price level is

$$p_t = E_{t-2}[m_t + v_t] + \frac{\phi}{1 + \phi} (E_{t-1}[m_t + v_t] - E_{t-2}[m_t + v_t])$$

As for equilibrium output:

$$y_t = m_t + v_t - E_{t-1}[m_t + v_t] + \frac{1}{1 + \phi} (E_{t-1}[m_t + v_t] - E_{t-2}[m_t + v_t])$$

Fischer model with demand shocks

Stabilization policy

- Let v_t follow a random walk ($v_t = v_{t-1} + \epsilon_t$, $\epsilon_t \sim WN(0, \sigma_\epsilon^2)$), and assume that monetary policy is given by the following rule

$$m_t = a_1 \epsilon_{t-1} + a_2 \epsilon_{t-2} + \dots + a_n \epsilon_{t-n} + \dots$$

- This rule is general in that it uses all the information available to the policymaker at time t (i.e., I_{t-1}). But it is special in having only linear terms
- Implicitly this presumes a particular form for society's preferences (we return to this issue after finding the optimal rule)

Fischer model with demand shocks

Stabilization policy

- We aim at solving for output under this monetary rule
- As a first step, let us re-shuffle the terms on the RHS of the output equation:

$$y_t = m_t + v_t - \frac{\phi}{1 + \phi} E_{t-1}[m_t + v_t] - \frac{1}{1 + \phi} E_{t-2}[m_t + v_t]$$

- Recall that

$$m_t + v_t = \underbrace{a_1 \epsilon_{t-1} + a_2 \epsilon_{t-2} + \dots + a_n \epsilon_{t-n} + \dots}_{=m_t} + \underbrace{v_{t-1} + \epsilon_t}_{=v_t}$$

Fischer model with demand shocks

Stabilization policy

- Let's work on the expectational terms:

-

$$\begin{aligned} E_{t-1}[m_t + v_t] &= E_{t-1}[v_{t-1} + \epsilon_t + a_1\epsilon_{t-1} + a_2\epsilon_{t-2} + \dots + a_n\epsilon_{t-n} + \dots] \\ &= v_{t-1} + a_1\epsilon_{t-1} + a_2\epsilon_{t-2} + \dots + a_n\epsilon_{t-n} + \dots \\ &= v_{t-1} + m_t \\ &= v_t - \epsilon_t + m_t \end{aligned}$$

-

$$\begin{aligned} E_{t-2}[m_t + v_t] &= E_{t-2}[v_{t-1} + \epsilon_t + a_1\epsilon_{t-1} + a_2\epsilon_{t-2} + \dots + a_n\epsilon_{t-n} + \dots] \\ &= E_{t-2}[v_{t-2} + \epsilon_{t-1} + \epsilon_t + a_1\epsilon_{t-1} + a_2\epsilon_{t-2} + \dots + a_n\epsilon_{t-n} + \dots] \\ &= v_{t-2} + a_2\epsilon_{t-2} + \dots + a_n\epsilon_{t-n} + \dots \\ &= \underbrace{v_{t-2}}_{=v_{t-1}-\epsilon_{t-1}} - a_1\epsilon_{t-1} + \underbrace{a_1\epsilon_{t-1} + a_2\epsilon_{t-2} + \dots + a_n\epsilon_{t-n} + \dots}_{=m_t} \\ &= \underbrace{v_{t-1}}_{=v_t-\epsilon_t} - \epsilon_{t-1} - a_1\epsilon_{t-1} + m_t \\ &= v_t - \epsilon_t - (1 + a_1)\epsilon_{t-1} + m_t \end{aligned}$$

Fischer model with demand shocks

Stabilization policy

- Therefore:

$$\begin{aligned}y_t &= m_t + v_t - \frac{\phi}{1 + \phi} (v_t - \epsilon_t + m_t) \\ &\quad - \frac{1}{1 + \phi} (v_t - \epsilon_t - (1 + a_1) \epsilon_{t-1} + m_t) \\ \rightarrow y_t &= \frac{\phi}{1 + \phi} \epsilon_t + \frac{1}{1 + \phi} (\epsilon_t + (1 + a_1) \epsilon_{t-1}) \\ \rightarrow y_t &= \epsilon_t + \frac{1 + a_1}{1 + \phi} \epsilon_{t-1}\end{aligned}$$

- Since ϵ_t and ϵ_{t-1} are uncorrelated, a policy that wants to minimize output volatility would choose

$$a_1 = -1$$

- This tells us the following on society's preferences and optimal policy: if we only dislike output volatility, then a linear policy rule is sufficient (crucial for the next lecture)

Stabilization policy in the Fischer model

- The optimal policy sets money (or another suitable tool) to offset anticipated non-policy shocks in the next period
- Other coefficients are irrelevant because past changes in aggregate demand are included in prices and thus have no real effects
- Thus, this model has persistence of shocks, but only for one period
- Taylor modifies the Fischer model by making chosen prices to be *fixed*, i.e. a firm setting prices at time t for periods t and $t + 1$ is forced to choose same prices for both periods
- This modification produces more persistence

An alternative application: Fixed prices (aka the Taylor model)

- Suppose now that individual prices are fixed for 3 periods and that price-setting is staggered, such that $1/3$ of the prices are set in period t at the level x_t , $1/3$ were set in period $t - 1$ at the level x_{t-1} , while a remaining $1/3$ were set in $t - 2$ at the level x_{t-2} . Thus, the aggregate price level equals

$$p_t = \frac{1}{3} (x_t + x_{t-1} + x_{t-2})$$

- Suppose that the (log) money supply follows a random walk:
 $m_t = m_{t-1} + \varepsilon_t$
- What kind of process characterizes aggregate inflation?

An alternative application: Fixed prices

- Assuming certainty equivalence:

$$x_t = \frac{1}{3} (p_t^* + E_t [p_{t+1}^*] + E_t [p_{t+2}^*])$$

with $p_t^* = m_t$ (we abstract from real rigidities, without loss of generality)

- Thus

$$x_t = \frac{1}{3} (m_t + E_t [m_{t+1}] + E_t [m_{t+2}])$$

Clearly, higher (contemporaneous and expected) money supply (m) increases the desired price, thereby x_t

An alternative application: Fixed prices

Derive an expression for aggregate price inflation:

$$\begin{aligned}\pi_t &= p_t - p_{t-1} \\ &= \frac{1}{3}(x_t + x_{t-1} + x_{t-2}) - \frac{1}{3}(x_{t-1} + x_{t-2} + x_{t-3}) \\ &= \frac{1}{3}x_t - \frac{1}{3}x_{t-3} \\ &= \frac{1}{3}\left(\frac{1}{3}(m_t + E_t[m_{t+1}] + E_t[m_{t+2}])\right) \\ &\quad - \frac{1}{3}\left(\frac{1}{3}(m_{t-3} + E_{t-3}[m_{t-2}] + E_{t-3}[m_{t-1}])\right) \\ &= \frac{1}{9}(m_t + E_t[m_{t+1}] + E_t[m_{t+2}]) \\ &\quad - \frac{1}{9}(m_{t-3} + E_{t-3}[m_{t-2}] + E_{t-3}[m_{t-1}])\end{aligned}$$

An alternative application: Fixed prices

Now, use the fact that $m_t = m_{t-1} + \varepsilon_t$, obtaining:

$$\begin{aligned}\pi_t &= \frac{1}{9} (m_t + E_t [m_t + \varepsilon_{t+1}] + E_t [m_{t+1} + \varepsilon_{t+2}]) - \frac{1}{9} (m_{t-3} + E_{t-3} [m_{t-3} + \varepsilon_{t-2}] + E_{t-3} [m_{t-2} + \varepsilon_{t-1}]) \\ &= \frac{1}{9} (m_t + E_t [m_t + \varepsilon_{t+1}] + E_t [m_t + \varepsilon_{t+1} + \varepsilon_{t+2}]) \\ &\quad - \frac{1}{9} (m_{t-3} + E_{t-3} [m_{t-3} + \varepsilon_{t-2}] + E_{t-3} [m_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1}]) \\ &= \frac{1}{3} (m_t - m_{t-3}) \\ &= \frac{1}{3} \left(\underbrace{m_{t-1} + \varepsilon_t}_{=m_t} - m_{t-3} \right) \\ &= \frac{1}{3} \left(\underbrace{m_{t-2} + \varepsilon_{t-1} + \varepsilon_t}_{=m_{t-1}} - m_{t-3} \right) \\ &= \frac{1}{3} \left(\underbrace{m_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t}_{=m_{t-2}} - m_{t-3} \right) \\ &= \frac{1}{3} (\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2})\end{aligned}$$

So, inflation follows an MA(2) process

Calvo model

- Calvo modifies the Taylor model by making price-setting stochastic
- Instead of firms knowing for sure that they are setting prices in odd or even periods, now *every period* firms are able to set new prices, but only with probability $0 < \alpha \leq 1$
- And prices must remain fixed until the firm is able to change them again

Calvo model

- The price level at time t is given by

$$p_t = \alpha x_t + (1 - \alpha)p_{t-1} \quad (6)$$

where x_t is the price chosen by firms that can update prices

- Note that x_t is not p_t^* (optimal price for period t) because firms must fix prices for, *a priori*, many periods (but do not know how many, exactly)

Calvo model: solution

- Optimal x_t is an *average of optimal p_t^* 's for all future periods*, with weights reflecting probability that a price chosen today is unchanged in the future, i.e.:

$$x_t = [(1 - \beta(1 - \alpha))] \sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t[p_{t+j}^*]$$

where $\beta \equiv$ discount factor

- Let's incorporate the term p_t^* :

$$x_t = [(1 - \beta(1 - \alpha))] p_t^* + [(1 - \beta(1 - \alpha))] \sum_{j=1}^{\infty} \beta^j (1 - \alpha)^j E_t[p_{t+j}^*]$$

Calvo model: solution

- Now:

$$x_t = [(1 - \beta(1 - \alpha))] p_t^* + \beta(1 - \alpha) \underbrace{\left[(1 - \beta(1 - \alpha)) \sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t[p_{t+1+j}^*] \right]}_{=E_t x_{t+1}}$$

- Subtracting p_t from each side of the equation above

$$\begin{aligned} x_t - p_t &= [1 - \beta(1 - \alpha)] (p_t^* - p_t) \\ &\quad + \beta(1 - \alpha) (E_t x_{t+1} - p_t) \end{aligned}$$

- Add and subtract p_{t-1} on the LHS of the equation above

$$\begin{aligned} (x_t - p_{t-1}) - (p_t - p_{t-1}) &= [1 - \beta(1 - \alpha)] (p_t^* - p_t) \\ &\quad + \beta(1 - \alpha) (E_t x_{t+1} - p_t) \end{aligned}$$

- From (6), the inflation rate is given by $\pi_t = \alpha(x_t - p_{t-1})$, thus:

$$x_t - p_{t-1} = \frac{\pi_t}{\alpha}$$

Calvo model: solution

- Using $p_t^* - p_t = \phi y_t$ the previous equations lead to

$$\pi_t = \frac{\alpha}{1 - \alpha} [1 - \beta(1 - \alpha)] \phi y_t + \beta E_t[\pi_{t+1}]$$

- This is the *new Keynesian Phillips curve*
- Notice how now inflation depends on expected future inflation, while in the Lucas model the relation was with expected current inflation

Calvo model: intuition

- We know the solution to this expectational difference equation:

$$\pi_t = \frac{\alpha\phi}{1-\alpha} [1 - \beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j E_t[y_{t+j}]$$

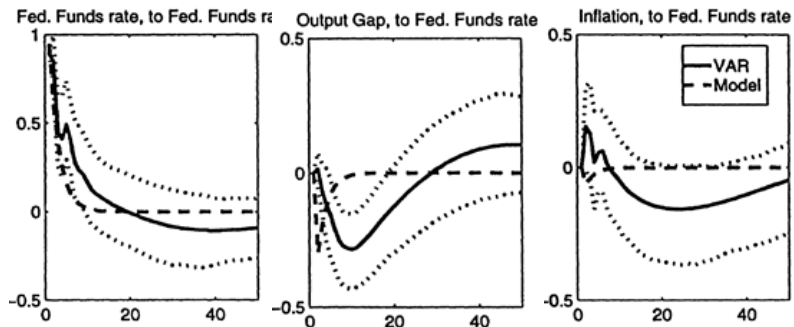
- Inflation today reflects expected future log-output realizations in deviation from steady state output

Persistence puzzle

- *Problem:* The standard NKPC fails to capture inflation persistence. In the simple model above the persistence of inflation derives from the persistence of real marginal costs (*inherited persistence*)
- *Empirical result:* When lagged inflation is added to the NKPC, it becomes strongly significant and the coefficient on expected inflation vanishes (Fuhrer, 1997)

Persistence puzzle

A quick look at the empirical evidence



Persistence puzzle

Alternative price-setting schemes

- A monetary policy shock has a long-lasting effect on inflation (as well as on output and prices), which is not captured by the baseline NKPC
- From an empirical viewpoint, intrinsic inertia has been contemplated
- Even if the size of the backward component of inflation (*intrinsic persistence*) is small, it is there and calls for an explanation
- Therefore, we need to extend the model to generate inflation persistence.

Popular specifications:

- Adaptive expectations
- Backward looking 'rule-of-thumb' price-setting behavior
- Partial indexation schemes

The dynamic New Keynesian Model

- A particular 'small-scale' DSGE model has received particular attention, and is now widely used by academics and central bankers alike
- As in most macro models, it features an AD block and an AS block, both of which can be derived from first principles
- AD block: the (log-linearized) consumption Euler equation (under CRRA utility)

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1})$$

also called 'optimizing' or 'dynamic' IS curve

- AS block: the New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- To close the model, we need a policy rule. For example:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + v_t$$