Macroeconomics III - Assignment 1

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October 2021

Identical competitive firms maximize the following profit function:

$$\pi^F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t K_t$$

Utility function is given as:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

Their budget constraint is:

$$c_t + a_{t+1} = w_t + (1 + r_t)a_t$$

a

We find the first-order conditions for firms maximization problem w.r.t capital and then labor:

$$\frac{\partial \pi^F}{\partial K_t} = \alpha K_t^{\alpha-1} L_t^{1-\alpha} - r_t = 0 \Leftrightarrow$$

$$\alpha K_t^{\alpha-1} L_t^{1-\alpha} = r_t \Leftrightarrow$$

$$\alpha k_t^{\alpha-1} = r_t \Leftrightarrow$$

$$\frac{\partial \pi^F}{\partial L_t} = (1-\alpha) K_t^{\alpha} L_t^{-\alpha} - w_t = 0 \Leftrightarrow$$

$$(1-\alpha) K_t^{\alpha} L_t^{-\alpha} = w_t \Leftrightarrow$$

$$(1-\alpha) k_t^{\alpha} = w_t \Leftrightarrow$$

When capital per capita increases, the marginal product of labor increases which in turn increases the households return to labor services (wage rate). However, when capital per capita increases, the marginal product of capital will fall, which in turn makes the decreases the return on savings (rental rate of capital). The complete opposite happens when capital per capita decreases.

b

We write the Lagrangian and find the first-order conditions

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} + \lambda_t[w_t + (1+r_t)a_t - c_t - a_{t+1}]$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t c_t^{-\theta} - \lambda_t = 0 \Leftrightarrow$$

$$\beta^t c_t^{-\theta} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -\lambda_t + \lambda_{t+1}(1+r_{t+1}) = 0 \Leftrightarrow$$

$$\lambda_{t+1}(1+r_{t+1}) = \lambda_t \Leftrightarrow$$

$$\beta^{t+1} c_{t+1}^{-\theta}(1+r_{t+1}) = \beta^t c_t^{-\theta} \Leftrightarrow$$

$$\beta^t(1+r_{t+1}) = \frac{c_t^{-\theta}}{c_{t+1}^{-\theta}} \Leftrightarrow$$

$$(\beta^t(1+r_{t+1}))^{\frac{1}{\theta}} = \frac{c_{t+1}}{c_t}$$

$\frac{1}{\theta}$ is the elasticity of intertemporal substitution of consumption. If $\theta \to \infty$ the household wants to smooth their consumption in both periods. When θ becomes arbitrarily small, the households wants to save up and use consumption in the later period. When $r_t \uparrow$, consumption in $c_{t+1} \uparrow$, because consumption in c_t becomes relatively more expensive compared to consumption in c_{t+1}. β is the discount factor of consumption. The larger the discount factor becomes, the household will become more patient regarding consumption in future periods.

c

In the steady state, we have $c_t = c_{t+1}$ and we have $k_t = k_{t+1} = k$, so the c-locus becomes:

$$(\beta^t(1 + r_{t+1}))^{\frac{1}{\theta}} = 1$$
$$\beta^t(1 + \alpha k_{t+1}^{\alpha-1}) = 1$$
$$\beta^t(1 + \alpha k^{\alpha-1}) = 1$$

K-locus is determined by the budget constraint.

In steady state $k_t = a_t$ and $k_{t+1} = a_{t+1}$ since there is no borrowing, which is given by $a_t = k_t + b_t$:

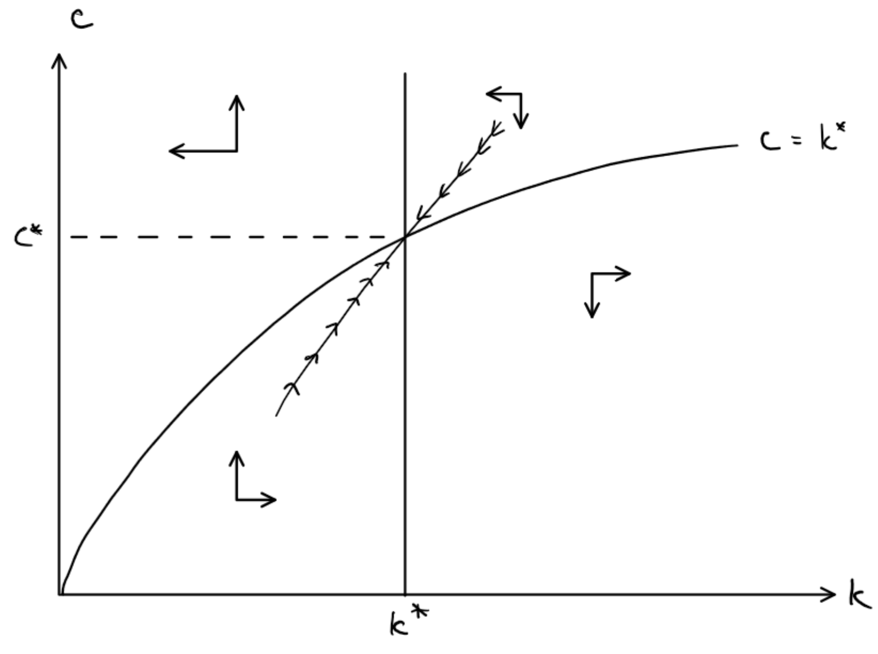
$$c_t + k_{t+1} = w_t + (1 + r_t)k_t \Leftrightarrow$$
$$c_t = w_t + (1 + r_t)k_t - k_{t+1} \Leftrightarrow$$
$$c_t = w_t + r_t k_t \Leftrightarrow$$

We now assume constant capital: $k_t = k_{t+1} = k$

Inserting the expressions for r_t and w_t

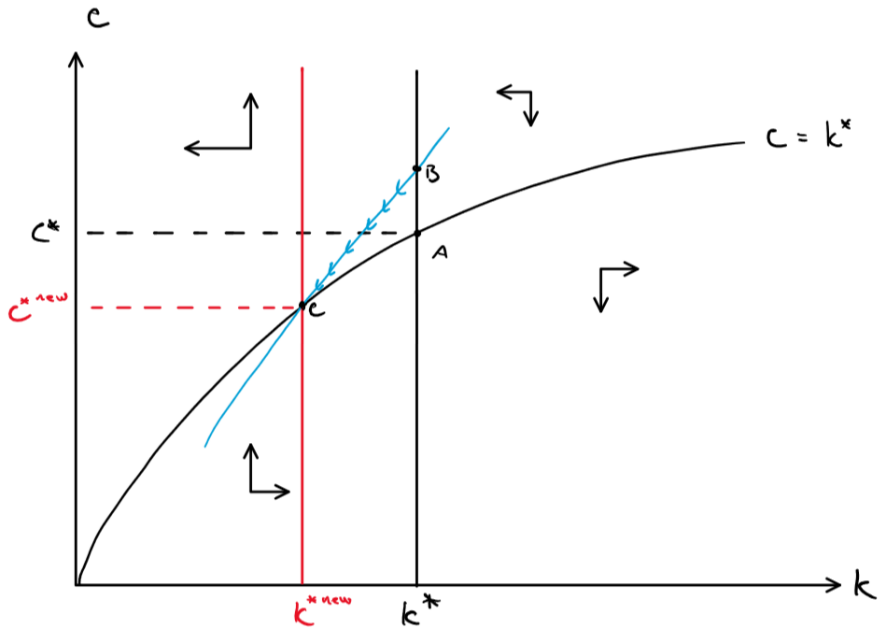
$$c = ((1 - \alpha)k^{\alpha}) + (\alpha k^{\alpha-1})k \Leftrightarrow c = (1 - \alpha)k^{\alpha} + \alpha k^{\alpha}$$
$$c = k^{\alpha}$$

The economy will converge towards (k^*, c^*) along the saddle path. If the economy is not on the saddle path, it will diverge.



d

When β is permanently lowered, the following will occur:



Before the shock, the economy is in the steady state 'A'. When the patience of the household decreases in period t, consumption in period t will increase. Therefore, we move to the point 'B'. Since consumption increased, savings must fall and the capital will decrease in period $t+1$. A fall in capital will result in a lower level of consumption, which in turn will decrease capital. This will keep repeating until we end up in the new steady state 'C'. Here we have both lower consumption and capital per capita.

e

The household's budget constraint now becomes:

$$c_t + a_{t+1} = w_t + [1 + r_t(1-\tau)a_t]$$

We find k-locus again with the new budget constraint.

$$c_t = w_t + (1-\tau)r_t k_t \Leftrightarrow$$

$$c_t = ((1-\alpha)k_t^{\alpha}) + (1-\tau)(\alpha k_t^{\alpha-1})k_t \Leftrightarrow$$

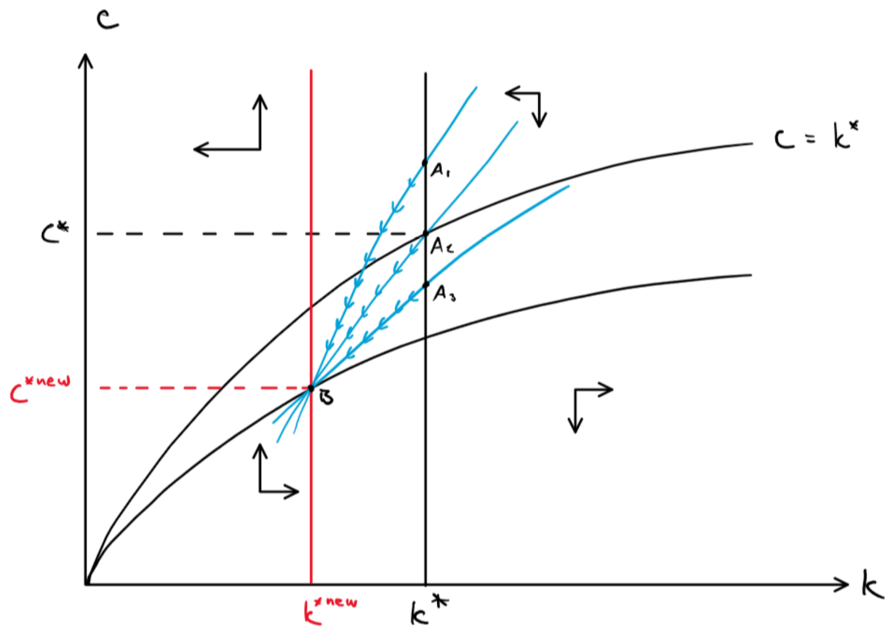
$$c_t = k_t^{\alpha} - \alpha k_t^{\alpha} + (1-\tau)\alpha k_t^{\alpha} \Leftrightarrow$$

$$c_t = k_t^{\alpha} - \tau\alpha k_t^{\alpha} \Leftrightarrow$$
$$c_t = (1 - \tau\alpha)k_t^{\alpha} \Leftrightarrow$$

c-locus will be

$$\beta^t(1 + (1 - \tau)r_{t+1}) = 1$$
$$\beta^t(1 + (1 - \tau)\alpha k^{\alpha-1}) = 1$$

We end up with the following phase diagram:



Following the introduction of the income tax, there are 2 contrasting effect. 1. Due to the income effect, consumption will increase in period t, as investing/saving will become less attractive following the tax. 2. Due to the substitution effect, savings will have to increase to maintain the same level of utility in future periods. Therefore the total short-run is ambiguous. However, the long run effect can only have both lower consumption and capital per capita.