# Macroeconomics III - Assignment 2

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## 1)

Utility for young individuals born i period t is:

$$U_t = c_{1t} + \beta \ln(C_{2t+1})$$

Production for firm i that hires labor and capital is given by:

$$Y_t^i = A(K_t^i)^\alpha (K_t N_t^i)^{1-\alpha}$$

The profit function is:

$$\pi = A\left(K_t^i\right)^\alpha \left(K_t N_t^i\right)^{1-\alpha} - w_t N_t^i - r_t K_t^i$$

### a)

Our maximization problem:

$$\max_{K_t^i, N_t^i} \pi = \pi = A\left(K_t^i\right)^\alpha \left(K_t N_t^i\right)^{1-\alpha} - w_t N_t^i - r_t K_t^i$$

wage rate:

$$\frac{\partial \pi}{\partial N_t^i} = (1-\alpha) A \left(K_t^i\right)^\alpha K_t^{1-\alpha} \left(N_t^i\right)^{-\alpha} - w_t = 0$$

$$\Rightarrow w_t = (1-\alpha) A \left(K_t^i\right)^\alpha K_t^{1-\alpha} \left(N_t^i\right)^{-\alpha}$$

We use that that total amount of firms must be n we can convert capital and labour for the individual firms. Therefore $K_t^i = K_t * n^{-1}$ and $N_t^i = N_t * n^{-1}$ Furthermore $N_t$ is normalized to 1:

$$\Rightarrow w_t = (1-\alpha) A K_t^\alpha n^{-\alpha} K_t^{1-\alpha} N_t^{-\alpha} n^\alpha = (1-\alpha) A K_t \Rightarrow w_t = (1-\alpha) A K_t$$

For rental rate:

$$\frac{\partial \pi}{\partial K_t^i} = \alpha A \left(K_t^i\right)^{\alpha-1} K_t^{1-\alpha} \left(N_t^i\right)^{1-\alpha} - r_t = 0$$

$$\Rightarrow r_t = \alpha A K_t^{\alpha-1} n^{\alpha-1} K_t^{1-\alpha} N^{1-\alpha} n^{1-\alpha} = A\alpha$$

Thus $r_t$ is independent of the capital stock.

## b

$$\max_{c_{1t}, c_{2t+1}} \qquad c_{1t} + \beta \ln(c_{2t+1})$$

$$s.t. \qquad c_{1t} = w_t - s_t - \tau w_t, c_{2t+1} = R_{t+1} s_t + \tau w_t$$

$w_{t+1}$ PAYG

Inserting the budget constraint into the max-problem:

$$\max_{s_t} \qquad w_t - s_t - \tau w_t + \beta \ln(R_{t+1} s_t + \tau w_t)$$

Taking the derivate wrt. $s_t$

$$\frac{\partial U_t}{\partial s_t} = 0 \Leftrightarrow$$

$$\frac{\beta R_{t+1}}{R_{t+1} s_t + \tau w_t} = 1 \Leftrightarrow$$

$$\beta R_{t+1} = R_{t+1} s_t + \tau w_t \Leftrightarrow$$

$w_{t+1}$

$$\frac{\beta R_{t+1} - \tau w_t}{R_{t+1}} = s_t$$

Savings are equal to capital accumulation due to no population growth $s_t = k_{t+1}$

We know that $R_t = 1 + r_t$ and $r_t = A\alpha$:

$$\frac{\beta(1 + A\alpha) - \tau w_{t}{}_{t+1}}{(1 + A\alpha)} = s_t = k_{t+1}$$

## c

The problem for young in period $t_0$:

$$\max_{c_{1t_0}, c_{2t_0+1}} \quad c_{1t_0} + \beta \ln(c_{2t_0+1})$$

$$s.t. \quad c_{1t_0} = w_t - s_{t_0} - \gamma \tau w_{t_0}(1-\epsilon), c_{2t_0+1} = (1+r)s_{t_0} + (1-\epsilon)\tau w_{t_0}$$

PAYG system

The max problem f or $t_0$ is::

$$\max_{s_t} \quad w_t - s_{t_0} - \gamma \tau w_{t_0}(1-\epsilon) + \beta \ln((1+r)s_{t_0} + (1-\epsilon)\tau w_{t_0})$$

Taking the derivative wrt. $s_{t_0}$

$$-1 + \frac{\beta(1+r)}{(1+r)s_{t_0} + (1-\epsilon)\tau w_{t_0}} = 0 \Leftrightarrow$$
$$\beta(1+r) = (1+r)s_{t_0} + (1-\epsilon)\tau w_{t_0} \Leftrightarrow$$
$$\frac{\beta(1+r) - (1-\epsilon)\tau w_{t_0}}{(1+r)} = s_{t_0}$$

We see that $\gamma$ has no effect on savings.

The government issues debt, which changes the capital accumulation:

↓ $\theta$ of which is being financed from abroad

$$k_{t+1} = s_t - (1-\gamma)\tau w_{t_0}(1-\epsilon)$$

So $K_{t+1} = S_t - (1-\theta)(1-\gamma)\tau w_{t_0}(1-\epsilon)$

When $\gamma = 0$ the capital accumulation becomes:

$$k_{t+1} = s_t - \tau w_{t_0}(1-\epsilon)$$

Capital accumulation decreases when $\gamma = 0$, and savings is unaffected. This is because the contributions to the social security system only financed from debt and not from the young. So, $\gamma = 0$ implies greater debt which in turn implies lower capital accumulation.

We see that when savings increase, capital increases too.

why? When $\gamma = 0$ the old's utility increases and the young becomes indifferent. The reform will have political support since it is Pareto optimal.

The problem for young individuals in period $t > t_0$

$$\max_{c_{1t}, c_{2t+1}} \quad c_{1t} + \beta \ln(c_{2t+1})$$

$$s.t. \quad c_{1t} = w_t - s_t - r(1-\gamma)\tau w_t(1-\epsilon) - \tau w_t(1-\epsilon)$$
$$c_{2t+1} = (1+r)s_t + (1-\epsilon)\tau w_t$$

$$\max_{s_t} \quad w_t - s_t - r(1-\gamma)\tau w_t(1-\epsilon) + \beta \ln((1+r)s_t + (1-\epsilon)\tau w_t)$$

Taking the derivative wrt. $s_{t_0+1}$

$$-1 + \frac{\beta(1+r)}{(1+r)s_t + (1-\epsilon)\tau w_t} = 0 \Leftrightarrow$$

$$\frac{\beta(1+r)}{(1+r)s_t + (1-\epsilon)\tau w_t} = 1 \Leftrightarrow$$

$$\frac{\beta(1+r) - (1-\epsilon)\tau w_t}{(1+r)} = s_t$$

Again, we see that savings is unaffected by $\gamma$. The capital accumulation is the same as in period $t_0$

what about the reform? How does it affect the savings?

$$k_{t+1} = s_t - (1-\gamma)\tau w_{t_0}(1-\epsilon) = s_t - \tau w_{t_0}(1-\epsilon)$$

So the effects are the same.

## d

In this case when $\gamma = 1$, we have that:

$$k_{t+1} = s_t - (1-\gamma)\tau w_{t_0}(1-\epsilon) = s_t$$

and:

$$\frac{\beta(1+r) - (1-\epsilon)\tau w_t}{(1+r)} = s_t$$

Savings are still unaffected by $\gamma$, and now the capital accumulation is only given by the savings.

We calculate the lifetime budget constraint in the PAYG for the young:

$$c_{1t} + \frac{c_{2t+1}}{1+r} = w_t - s_t - \tau w_t + \frac{R_{t+1}s_t + \tau w_t}{1+r} \quad t+1$$

$$= w_t + \frac{\tau w_t - (1+r)\tau w_t}{1+r} = w_t + \frac{r\tau w_t}{1+r}$$

When the social security system is reduced, we get the following lifetime budget constraint for the young:

$$c_{1t} + \frac{c_{2t+1}}{1+r} = w_t - s_{t_0} - \tau w_{t_0}(1-\epsilon) + \frac{(1+r)s_{t_0} + (1-\epsilon)\tau w_{t_0}}{1+r}$$



welfare with reform

$$C_{1t} = [1 - \gamma(1-\epsilon)\tau] W_t - S_t$$

$$C_{2t+1} = (1+r)S_t + (1-\epsilon)\} W_{t_0+1}$$

$$\Rightarrow C_{1t} + \frac{C_{2t+1}}{1+r} = W_t - \tau(1-\epsilon)\left[\gamma W_{t_0} - \frac{W_{t_0+1}}{1+r}\right]$$

regardless of the value of $\gamma$ (even if $\gamma = 1$) they'll support the reform

$$\text{VS} \quad C_{1t} + \frac{C_{2t+1}}{1+r} = W_t - \tau\left[W_{t_0} - \frac{W_{t_0+1}}{1+r}\right]$$

without reform

$$= w_t - \tau w_{t_0}(1-\epsilon) + \frac{(1-\epsilon)\tau w_{t_0}}{1+r}$$
$$= w_t + \frac{(1-\epsilon)\tau w_{t_0} - (1+r)\tau w_{t_0}(1-\epsilon)}{1+r}$$
$$= w_t + \frac{r\tau w_{t_0}(1-\epsilon)}{1+r}$$

The young will only vote to reduce the social security system when:

$$w_t + \frac{r\tau w_t(1-\epsilon)}{1+r} > w_t + \frac{r\tau w_t}{1+r}$$
$$r\tau w_t(1-\epsilon) > r\tau w_t \Leftrightarrow 1-\epsilon > 1 \Leftrightarrow 0 > \epsilon$$

This will never hold, since $\epsilon > 0$. The young will get a lower lifetime budget, so they will choose not to vote for the reform.

## e

The old savings rate was:

$$\frac{\beta(1+r) - \tau w_t}{(1+r)} = s_t$$

Their new savings rate is:

$$\frac{\beta(1+r) - (1-\epsilon)\tau w_t}{(1+r)} = s_t$$

Reducing the social security system will lead to a higher savings rate. This is caused by the lower welfare for the old, so the young will have a higher incentive to save in period $t$ and consume in $t+1$.