Macroeconomics III - Assignment 3

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(a)

We will find p^f. We plug the price level into the flexible prices:

$$p^f = (1-\phi)p + \phi m$$

We have two situations. One where $(m - E(m))^2 \leq \sigma^2$ and we get:

$$p = \alpha_1 p^r + \alpha_2 p^r + (1 - \alpha_1 - \alpha_2)p^f$$
$$p = (\alpha_1 + \alpha_2)p^r + (1 - \alpha_1 - \alpha_2)p^f$$

And one where $(m - E(m))^2 > \sigma^2$:

$$p = \alpha_1 p^r + \alpha_2 p^f + (1 - \alpha_1 - \alpha_2)p^f$$
$$p = \alpha_1 p^r + (1 - \alpha_1)p^f$$

We can write $\alpha = \alpha_1 + \alpha_2$ if $(m - E(m))^2 \leq \sigma^2$ and $\alpha = \alpha_1$ if $(m - E(m))^2 > \sigma^2$. Therefore, we get:

$$p = \alpha p^r + (1-\alpha)p^f$$

Inserting in p^f

$$p^f = (1-\phi)[\alpha p^r + (1-\alpha)p^f] + \phi m$$
$$p^f(1 - (1-\phi)(1-\alpha)) = (1-\phi)\alpha p^r + \phi m$$
$$p^f(\phi + (1-\phi)\alpha) = (\phi + (1-\phi)\alpha)p^r + \phi(m - p^r)$$
$$p^f = p^r + \frac{\phi}{\phi + (1-\phi)\alpha}(m - p^r)$$

Now we find p^r:

$$p^r = (1-\phi)E[p] + \phi E[m]$$

Inserting the price level:

$$p^r = (1-\phi)E[\alpha p^r + (1-\alpha)p^f] + \phi E[m]$$

$$p^r = (1-\phi)E[\alpha p^r + (1-\alpha)[p^r + \frac{\phi}{\phi + (1-\phi)\alpha}(m - p^r)]] + \phi E[m]$$

We remember that the rigid price firms set their prices before m is known, hence $E[p^r] = p^r$:

$$p^r = (1-\phi)\alpha p^r + (1-\phi)(1-\alpha)p^r + \frac{(1-\phi)\phi}{\phi + (1-\phi)\alpha}(E[m] - p^r) + \phi E[m]$$

$$p^r = \alpha p^r - \phi\alpha p^r + (1-\phi)(p^r - \alpha p^r) + \frac{(1-\phi)\phi}{\phi + (1-\phi)\alpha}(E[m] - p^r) + \phi E[m]$$

$$p^r = \alpha p^r - \phi\alpha p^r + p^r - \phi p^r + \phi\alpha p^r - \alpha p^r + \frac{(1-\phi)\phi}{\phi + (1-\phi)\alpha}(E[m] - p^r) + \phi E[m]$$

$$0 = -\phi p^r + \frac{(1-\phi)\phi}{\phi + (1-\phi)\alpha}(E[m] - p^r) + \phi E[m]$$

$$0 = \frac{(1-\phi)\phi}{\phi + (1-\phi)\alpha}(E[m] - p^r) + \phi(E[m] - p^r)$$

$$0 = (\phi + \frac{(1-\phi)\phi}{\phi + (1-\phi)\alpha})(E[m] - p^r))$$

For this to be equal to zero, the last parentheses must be zero. Therefore, we get:

$$E[m] = p^r$$

For p^{rf}, we have two situations:

$$p^{rf} = \begin{cases} E[m] & (m - E(m))^2 \leq \sigma^2 \\ p^r + \frac{\phi}{\phi + (1-\phi)\alpha_1}(m - p^r) & (m - E(m))^2 > \sigma^2 \end{cases}$$

(b)

We insert the rigid firm prices into the aggregate level prices:

$$p = \alpha p^r + (1-\alpha)[p^r + \frac{\phi}{\phi + (1-\phi)\alpha}(m - p^r)]$$

$$p = p^r + \frac{(1-\alpha)\phi}{\phi + (1-\phi)\alpha}(m - p^r)$$

Inserting the expression for p^r

$$p = E[m] + \frac{(1-\alpha)\phi}{\phi + (1-\phi)\alpha}(m - E[m])$$

We insert the prices into the output expression:

$$y = m - E[m] + \frac{(1-\alpha)\phi}{\phi + (1-\phi)\alpha}(m - E[m])$$

$$y = (1 - \frac{(1-\alpha)\phi}{\phi + (1-\phi)\alpha}(m - E[m])$$

$$y = (\frac{\phi + (1-\phi)\alpha}{\phi + (1-\phi)\alpha} - \frac{(1-\alpha)\phi}{\phi + (1-\phi)\alpha}(m - E[m])))$$

$$y = (\frac{\phi + (1-\phi)\alpha}{\phi + (1-\phi)\alpha} - \frac{(1-\alpha)\phi}{\phi + (1-\phi)\alpha}(m - E[m]))$$

$$y = \frac{\phi + (1-\phi)\alpha - (1-\alpha)\phi}{\phi + (1-\phi)\alpha}(m - E[m])$$

$$y = \frac{\alpha}{\phi + (1-\phi)\alpha}(m - E[m]))$$

We see that, for an expected change in m it holds that $E[m] = m$. Hence, an expected change in m does not change the output. Intuitively, higher money supply only affects the price level if the increase is expected. A higher expected money supply will. An expected change in the money supply will result in every agent raising their price which in turns lowers the output.

(c)

For an unanticipated change in m, it holds that $E[m] \neq m$. Therefore, an increase in m leads to an increase in y since, $m > E[m]$, which makes the last parentheses positive. An increase in m, can be seen as a short term increase in demand. And since only flexible firms can adopt the correct prices, the price level will be lower than optimally, which results in higher demand.

$$y = \frac{\alpha}{\phi + (1-\phi)\alpha}(m - E[m]))$$

(d)

If $(m - E(m))^2 \leq \sigma^2$ an unanticipated change in m will have the following effect:

$$\frac{\partial y}{\partial m} = \frac{\alpha_1 + \alpha_2}{\phi + (1-\phi)\alpha_1 + (1-\phi)\alpha_2}$$

If $(m - E(m))^2 > \sigma^2$ an unanticipated change in m will have the following effect:

$$\frac{\partial y}{\partial m} = \frac{\alpha_1}{\phi + (1-\phi)\alpha_1}$$

To see where the change in m will have the greatest effect we model the following equation, where a small shock is bigger than a larger shock:

$$\frac{\alpha_1 + \alpha_2}{\phi + (1-\phi)\alpha_1 + (1-\phi)\alpha_2} \geq \frac{\alpha_1}{\phi + (1-\phi)\alpha_1}$$

$$(\alpha_1 + \alpha_2)(\phi + (1-\phi)\alpha_1) \geq \alpha_1(\phi + (1-\phi)\alpha_1 + (1-\phi)\alpha_2)$$

$$\alpha_1(\phi + (1-\phi)\alpha_1) + \alpha_2(\phi + (1-\phi)\alpha_1) \geq \alpha_1\phi + (1-\phi)\alpha_1^2 + (1-\phi)\alpha_2\alpha_1$$

$$\phi\alpha_1 + (1-\phi)\alpha_1^2 + \phi\alpha_2 + (1-\phi)\alpha_1\alpha_2 \geq \alpha_1\phi + (1-\phi)\alpha_1^2 + (1-\phi)\alpha_2\alpha_1$$

$$\phi\alpha_2 \geq 0$$

This is obviously true since $\phi, \alpha_2 \geq 0$. This means that an unanticipated change in m will have the highest effect for small shocks.

When we have a small shock to m, the rigid-flex firms will not change their prices. A large fraction of the firms will not change their prices, and the effect from an increase in the money supply will result in a small increase in prices. Therefore, output will increase more compared to the situation where a shock is large.

(e)

The output becomes more responsive to an unanticipated small shock in m, but it has no effect on a large shock to m. The larger α_2 is, the larger the fraction of rigid-flex firms there are compared to flexible firms. This again results in more firms not adjusting their prices when there is an increase in the money supply, which in turn gives an increase to output.