

Microeconomics III: Problem Set 1^a

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 $[^]a{\rm Slides}$ created with reservation for possible errors.

Outline

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Overview of the course

Exam example

PS1, Ex. 2: Basics of game theory

PS1, Ex. 3: IESDS

PS1, Ex. 4: The Travelers' Dilemma

PS1, Ex. 5: IESDS

PS1, Ex. 6: The higher number wins

PS1, Ex. 7: Three player game

Preparation for exercise classes

Motivation

From the course description:

- This course furthers the *introduction* of game theory and its applications in economic models.
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- The student who successfully completes the course will learn the basics of game theory and will be enabled to work further with advanced game theory.
- The student will also learn how economic problems involving strategic situations can be *modeled* using game theory, as well as how these models are *solved*.
- 4. The course intention is that the student *becomes able to work with* modern economic theory, for instance within the areas of industrial organization, macroeconomics, international economics, labor economics, public economics, political economics and financial economics.

Courses where game theory is central:

- Mechanism Design
- Contract Theory
- Auctions Theory and Practice, Incentives and Organizations
- Industrial Organization
- Advanced Industrial Organization
- Strategic Management
- Advanced Strategic Management
- Behavioral Finance (F)
- Foundations of Behavioral Economics
- Behavioral and Experimental Economics (summerschool)
- Science of Behavior Change

Courses where game theory plays a part:

- Public Finance (taxation)
- Labour Economics
- Health Economics
- Political Economics
- Advanced Development Economics -Micro Aspects

Strategic, logical thinking is also useful for macroeconomists and econometricians

Overview of the course

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Form:

- 13 lectures + exam
- 12 problem sets in 12 exercise classes
- 1 exam prep class
- 1 question session before the exam
- Possibly an exam training exercise class in week 21
- 3 mandatory assignments delivered through Absalon October 2nd, October 30th, November 27th
- All 3 assignments must be passed, handing in automatically gives a redo.
- Make up assignment end of semester, will require turning in both assignment 4, and the one you missed.

Course content

- 1. Static games with complete information (PS 1-3)
- 2. Dynamic games with complete information (PS 4-6)
- 3. Static games with incomplete information (PS 3, 8-10)
- 4. Dynamic games with incomplete information (PS 6-7, 10-12)

Exam form:

 Most likely Two hours without aids on Peter Bangs Vej 36

Content:

- Cook-book solutions trained in the problem sets
- Reflection and discussion

Exam form:

 Two hours without aids on Peter Bangs Vej 36

Content:

- Cook-book solutions from the problem sets
- Reeflection and discussion

Important to complete all problem sets but also to attend the lectures and read the curriculum:

- The course can seem deceptively simple at first. Leading people to take a break for a few lectures, only to come back and not be able to follow along anymore.
- At the exam, students tend to show a general lack in the ability to *interpret, explain, and give examples* from the real world. This is something you will mainly get from the lectures.

Students also tend to feel a time pressure which leads to errors, where possibly we will train quick solution methods.

From the course description:

Knowledge:

- Formally state the definition of a game and explain the key differences between games of different types.
- In detail account for the equilibrium (solution) concepts that are relevant for these games (Nash Equilibrium, Subgame Perfect Nash Equilibrium, Bayes-Nash Equilibrium, Perfect Bayesian Equilibrium).
- Identify a number of special games and particular issues associated with them, such as repeated games (including infinitely repeated games), auctions and signaling games.

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Skills:

- 1. Explicitly *solve* for the equilibria of these games.
- 2. *Explain* the relevant steps in the reasoning of the solution.
- 3. *Interpret* the outcomes of the analysis.
- Apply equilibrium *refinements* and *discuss* the solution concepts

Competencies:

- Analyze strategic situations by modeling them as formal games.
- Set up, prove, analyze and apply the theories and methods used in the course in an *independent* manner.
- Evaluate and discuss the crucial assumptions underlying the theory.

Exam example

Example from the exam Autumn 2018

- "The reason that players cannot achieve a good outcome in the prisoner's dilemma is that they cannot communicate." True or false? Explain in 2-3 sentences.
- 1.c "Iterated Elimination of Strictly Dominated Strategies never eliminates a Nash Equilibrium" True or false? Explain in 2-3 sentences.
- 1.d You are writing your dating app profile and want to signal that you are adventurous. Give an example of a signal that is not credible and an example that is credible and explain the reasons why.

Take 5 min to discuss the questions with your neighbor(s)

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1.a True or false? Why?

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- 1.d Credible? Not credible? Difference?

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- True! Nash equilibrium is a refinement. However, this does not hold for weakly dominated strategies.
- 1.d Credible signals: Show a picture of you skydiving, swimming with sharks etc. Not credible: Write that you are adventurous or only claim that you have been skydiving etc. There needs to be a differential cost that makes it affordable for those with a hidden desirable trait (being adventurous), not affordable for those without this trait.

There is a cost to the signal: Those with the desirable trait are more likely to send the signal. Those who exhibit the signal are more desirable.

PS1, Ex. 2: Basics of game theory

What is game theory and why do we do it? To answer this, briefly discuss the following questions:

- (a) What are the ingredients of a (normal form) game?
- (b) How do we analyze games?
- (c) Why do you think it is practical to analyze problems as games?

Take 5 min. to discuss it with your neighbor(s)

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- (a) A normal form game consists of:
 - 1. The set of players i
 - 2. The possible strategy sets $S_i \in \{s_1, s_2, ..., s_n\}$ for each player *i*
 - Each players utility (payoff) function u_i(s₁, s₂, ..., s_n)

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- (b) Define a solution concept, state the assumptions it relies on, and its possible limitations.
 E.g. Iterative Elimination of Strictly Dominated Strategies (IESDS) requires common knowledge of rationality but is not always sufficient to find the Nash Equilibrium.
- (c) Complex situations can be analyzed in an unambiguous way through modelling them as games and applying logic.

PS1, Ex. 3: IESDS

Take 5 min. to solve it on your own or with your neighbor(s)

	t_1	t ₂	t ₃
s_1	5, 0	3, 3	1, 1
<i>s</i> ₂	3, 4	2, 2	3, 1
s 3	2, 2	1, 1	0, 5

	t_1	t ₂	t ₃
s 1	5 , 0	3 , 3	1 , 1
s 2	3 , 4	2 , 2	3 , 1
s 3	<mark>2</mark> , 2	1 , 1	<mark>0</mark> , 5

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game:

	t_1	t ₂	t ₃
<i>s</i> ₁	5, 0	3, 3	1, 1
s 2	3, 4	2, 2	3, 1

	t_1	t ₂	t ₃
<i>s</i> 1	5 , 0	3 , 3	1 , 1
<i>s</i> ₂	3 , 4	<mark>2</mark> , 2	3 , 1
5 3	<mark>2, 2</mark>	<mark>1, 1</mark>	<mark>0, 5</mark>

Giving us a new reduced form game:

	$ t_1 $	t ₂	
s_1	5, 0	3, 3	
<i>s</i> ₂	3, 4	2, 2	

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game:

	$ t_1$	t	2	t3
s_1	5,	0 3,	3 1	, 1
s ₂	3,	4 2,	2 3	, 1

Player 2: t_3 is strictly dominated by t_2 and can be eliminated.

	t_1	t ₂	t ₃
s 1	5 , 0	3 , 3	1 , 1
s 2	3, 4	<mark>2</mark> , 2	3 , 1
53	<mark>2, 2</mark>	<mark>1, 1</mark>	<mark>0, 5</mark>

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game:

	t_1	t ₂	t3
s_1	5, 0	3, <mark>3</mark>	1, 1
<i>s</i> ₂	3, 4	2, 2	3, 1

Player 2: t_3 is strictly dominated by t_2 and can be eliminated.

Giving us a new reduced form game:

	t_1	t ₂
<i>s</i> 1	<mark>5</mark> , 0	<mark>3</mark> , 3
s 2	<mark>3</mark> , 4	<mark>2</mark> , 2

Player 1: s_2 is strictly dominated by s_1 and is eliminated. Reduced form game:

	t_1	t ₂	t ₃
s 1	5 , 0	3 , 3	1 , 1
s 2	3 , 4	2 , 2	3 , 1
53	<mark>2, 2</mark>	1, 1	0, 5

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game:

	t_1	<i>t</i> ₂	t3
<i>s</i> ₁	5, 0	3, <mark>3</mark>	1, 1
s 2	3, 4	2, 2	3, 1

Player 2: t_3 is strictly dominated by t_2 and can be eliminated.

Giving us a new reduced form game:

	t_1	t ₂
<i>s</i> 1	5 , 0	3 , 3
s 2	3, 4	<mark>2, 2</mark>

Player 1: s_2 is strictly dominated by s_1 and is eliminated. Reduced form game:

Player 2: t_1 is strictly dominated by t_2 and is eliminated. I.e. IESDS provides the unique strategy profile (s_1, t_2), implying that this is also the Nash Equilibrium:

$$\begin{array}{c|c} t_2 \\ \hline s_1 & 3, 3 \end{array}$$

PS1, Ex. 4: The Travelers' Dilemma

The Travelers' Dilemma:

"An airline loses two suitcases belonging to two different travelers. Both suitcases look identical and contain identical items. An airline manager tasked to settle the claims of both travelers explains that the airline is liable for a maximum of \$100 per suitcase, and in order to determine an honest appraised value of the antiques the manager separates both travelers so they can't confer, and asks them to write down the amount of their value no less than \$0 and no larger than \$100. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount

However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with the following: \$1 extra will be paid to the traveler who wrote down the lower value and a \$1 fine imposed on the person who wrote down the higher amount."

- (a) Write down the normal form of this game: players, strategy sets, payoffs
- (b) Can you solve this game by IESDS?
- (c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

Hint: The game does not work with discrete increments of 1 dollar in bids, but you can use increments of one cent

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- (c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

- (a) A normal form game consists of:
 - 1. The set of players: Traveler 1 and Traveler 2.
 - 2. Strategy sets: $S_i = \{0; 0.01; ...; 99.99; 100\}$ for i = 1; 2
 - 3. Payoffs for player $i \neq j$:

$$u_i(s_i, s_j) = \begin{cases} s_i & \text{if } s_i = s_j \\ s_i + 1 & \text{if } s_i < s_j \\ s_j - 1 & \text{if } s_i > s_j \end{cases}$$

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(b) No, as no strategy is always dominated by one other strategy no matter what the other traveler plays.

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- (b) No, as no strategy is always dominated by one other strategy no matter what the other traveler plays.
- (c) Given common knowledge of rationality each traveler will avoid getting "underbid" by the other, i.e. the Nash Equilibrium is $s_i, s_j = (0, 0)$ as there is no incentive to deviate.

PS1, Ex. 5: IESDS

Solve these games by iterative elimination of strictly dominated strategies:

$\begin{vmatrix} t_1 & t_2 & t_3 \end{vmatrix}$	t_1 t_2 t_3
s ₁ 5, 0 2, 3 1, 1	s ₁ 5, 0 2, 3 1, 1
s ₂ 2, 4 2, 2 3, 1	s ₂ 2, 4 2, 2 3, 1
s ₃ 2, 2 1, 1 0, 5	s ₃ 2, 2 1, 1 1, 5

Hint: A strategy only need to be strictly dominated by one strategy to be eliminated by IESDS, if one of the outcomes makes the payoff for two strategies equal to each other (fx. s3 and s2 if t1), then the strategy is not strictly dominated, but it can be weakly dominated, which will be introduced next lecture.

Solve these games by iterative elimination of strictly dominated strategies:

	t_1	t ₂	t ₃
<i>s</i> ₁	5,0	2, 3	1, 1
s 2	2, 4	2, 2	3, 1
53	2, 2	1,-1	0, 5

- 1. Player 1: s_3 is strictly dominated by s_1 and is eliminated
- Player 2: After eliminating s₃, t₃ is strictly dominated by t₂ and is eliminated, giving us the reduced form game:

	t_1	t_2
s_1	5,0	2, 3
s ₂	2, 4	2, 2

	t_1	t ₂	t ₃
<i>s</i> ₁	5, 0	2, 3	1, 1
<i>s</i> ₂	2, 4	2, 2	3, 1
5 3	2, 2	1, 1	1, 5

PS1, Ex. 6: The higher number wins

Mikael and Jonas are playing a game instead of working. The game has the following rules: Both secretly pick a (natural) number between 1 and 5. Then they reveal the numbers to each other. If both have picked the same number, nobody gets anything. If Jonas' number is higher than Mikael's number, Mikael has to pay Jonas 1 kr. If Mikael's number is higher than Jonas', Jonas has to pay 10 kr. to Mikael

- (a) Does this game seem fair to you?
- (b) Write the game in bimatrix form.
- (c) Are there any strictly dominated strategies? Solve the game by iterated elimination of strictly dominated strategies.
- (d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

Take 10 min. to answer the questions on your own or with your neighbor(s)

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- (d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

- (a) How do you define "fair"?Symmetric payoffs and/or equal chance of winning?
- (b) In bimatrix form:

	"1"	"2"	"3"	"4"	"5"
"1"	0, 0	-1, 1	-1, 1	-1, 1	-1, 1
"2"	10, -10	0, 0	-1, 1	-1, 1	-1, 1
"3"	10, -10	10, -10	0, 0	-1, 1	-1, 1
"4"	10, -10	10, -10	10, -10	0, 0	-1, 1
"5"	10, -10	10, -10	10, -10	10, -10	0, 0

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- (b) Write the game in bimatrix form.
- (c) Are there any strictly dominated strategies? Solve the game by iterated elimination of strictly dominated strategies.
- (d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

		"1"		"2"	"3"		"4"		"5"
"1"		0,0		-1, 1	-1, 1		-1, 1		-1, 1
"2"		10, -10		0, 0	-1, 1		-1, 1		-1, 1
"3"		10, -10		10, -10	0, 0		-1, 1		-1, 1
"4"		10, -10	1	10, -10	10, -10		0, 0		-1, 1
"5"		10, -10		10, -10	10, -10		10, -10		0, 0

- (c) In the initial game playing "2", "3", or "4" is only weakly dominated by playing "5". However, "1" is strictly dominated by "5". After removing "1" for each player, "2" is strictly dominated by "5" and is removed for each player as well. Continue till only ("5","5") survive IESDS.
- (d) Both IESDS and Nash Equilibrium require common knowledge of rationality. If the assumption holds, ("5","5") is the outcome.

PS1, Ex. 7: Three player game

We can also write games with more than two players. Consider the game below where player 1 chooses the bi-matrix (A or B), player 2 chooses the row (C or D), and player 3 chooses the column (E or F). In each cell, the first number gives the payoff of Player 1, the second number the payoff of Player 2, and the third number the payoff of Player 3.

E F	E	F
C 0, 2, 2 2, 1, 1	C 1, 0, 1 3	3, 1, 2
D 0, 1, 1 3, 0, 0	D 1, 1, 0 5	5, 2, 1
А	В	

Find the pure strategy profiles that survive iterated elimination of strictly dominated strategies. (10 min.)



1st step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:

	Е	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1
	В	



1st step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:



 2^{nd} step: Player 2: After matrix A is eliminated, C is strictly dominated by D and we eliminate C.



1st step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:



 2^{nd} step: Player 2: After matrix A is eliminated, C is strictly dominated by D and we eliminate C.

 3^{rd} step: Player 3: After matrix A and row C is eliminated, E (payoff = 0) is strictly dominated by F (payoff = 1) and we eliminate E.

The unique pure strategy profile that survives IESDS is (B; D; F).

Preparation for exercise classes

To get through all problem sets you need to show up prepared:

- After reading the lecture slides, take at least 60 minutes to try and solve the problem set without any aids (both a and b).
- The resulting frustration will signal to your brain which parts are important to remember, and thus help with memorisation which is vital for the exam.