Microeconomics III: Problem Set 8[a]

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April 2 2020

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[a]Slides created for exercise class with reservation for possible errors.

Outline

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game:

PS8, Ex. 2 (A): Static Bayesian game (Bayesian Nash Equilibria)

PS8, Ex. 3: Static Bayesian game (Bayesian Nash Equilibria)

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

PS8, Ex. 4: The dating game (Bayesian Nash Equilibria)

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game:

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game:

Consider the following game G:

		Player 2		
		X	Y	Z
Player 1	A	6, 6	0, 8	0, 0
	B	7, 1	2, 2	1, 1
	C	0, 0	1, 1	4, 5

Suppose that G is repeated infinitely many times, so that we have G(1, ∞). Define trigger strategies such that the outcome of all stages is (A,X). Find the smallest value of δ such that these strategies constitute a SPNE.

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game - exam answer)

Suppose that G is repeated infinitely many times, so that we have G(1, ∞). Define trigger strategies such that the outcome of all stages is (A,X). Find the smallest value of δ such that these strategies constitute a SPNE.

Trigger strategies such that the outcome of all stages of the game is (A,X) are possible using respectively B,Y or C,Z as the threats. Since the threats B,Y will make the SPNE possible for the smallest δ, I will use B,Y in the trigger strategies i define:

1. Trigger strategy $P1$: In the 1st turn, play A. In every subsequent turn, if outcome from every previous turn was (A,X), play A, otherwise play B.
2. Trigger strategy $P2$: In the 1st turn, play X. In every subsequent turn, if outcome from every previous turn was (A,X), play X, otherwise play Y.

Player 2 has the highest incentive to deviate, so I only examine player 2's incentive to deviate. In order to find the lowest δ to secure cooperation, I set up the inequality for which the payoff for cooperation is higher than the payoff for deviating:

$$\frac{6}{1-\delta} \geq 8 + \frac{2\delta}{1-\delta} \Leftrightarrow$$
$$6 \geq 8 - 8\delta + 2\delta \Leftrightarrow$$
$$\delta \geq \frac{1}{3}$$

$\delta = \frac{1}{3}$ is the smallest value for which the strategies constitute a SPNE.

PS8, Ex. 2 (A): Static Bayesian game (Bayesian Nash Equilibria)

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Consider the following static game, where a is a real number:

	L	R
U	2, 1	0, a
D	0, 1	1, a

(a) Suppose that $a = 2$. Does any player have a dominant strategy? What about when $a = -2$?

(b) Now assume that player 2 knows the value of a, but player 1 only knows that $a = 2$ with probability 0.5 and $a = -2$ with probability 0.5. Explain how this situation can be modeled as a Bayesian game, describing the players, their action spaces, type spaces, beliefs and payoff functions.

(c) Find the Bayes-Nash equilibrium of the game described in (b).

PS8, Ex. 2.a (A): Static Bayesian game (Bayesian Nash Equilibria)

(a) Suppose that $a = 2$. Does any player have a dominant strategy? What about when $a = -2$?

(a) The value of a affects P2's payoff:

$a = 2$:

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

$a = -2$:

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

PS8, Ex. 2.a (A): Static Bayesian game (Bayesian Nash Equilibria)

(a) Suppose that $a = 2$. Does any player have a dominant strategy? What about when $a = -2$?

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

$a = 2:$

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

$a = -2:$

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

PS8, Ex. 2.b (A): Static Bayesian game (Bayesian Nash Equilibria)

(b) Now assume that player 2 knows the value of a, but player 1 only knows that $a = 2$ with probability 0.5 and $a = -2$ with probability 0.5. Explain how this situation can be modeled as a Bayesian game, describing the players, their action spaces, type spaces, beliefs and payoff functions.

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

$a = 2:$

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

$a = -2:$

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

(b) Now assume that player 2 knows the value of a, but player 1 only knows that $a = 2$ with probability 0.5 and $a = -2$ with probability 0.5. Explain how this situation can be modeled as a Bayesian game, describing the players, their action spaces, type spaces, beliefs and payoff functions.

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

(b) This can be modelled as a Bayesian game since P2 has two types (where he strongly prefers L and R respectively) and P1 has a belief about the distribution of these types (each happen $\frac{1}{2}$ of the time.).

$a = 2$:

	L	R
U	2, 1	0, 2
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$a = -2$:

	L	R
U	2, 1	0, -2
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PS8, Ex. 2.b (A): Static Bayesian game (Bayesian Nash Equilibria)

(b) Now assume that player 2 knows the value of a, but player 1 only knows that $a = 2$ with probability 0.5 and $a = -2$ with probability 0.5. Explain how this situation can be modeled as a Bayesian game, describing the players, their action spaces, type spaces, beliefs and payoff functions.

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

(b) This can be modelled as a Bayesian game since P2 has two types (where he strongly prefers L and R respectively) and P1 has a belief about the distribution of these types (each happen $\frac{1}{2}$ of the time.).

Players: P1, P2

Action sp.: $A_1 = (U, D), A_2 = (L, R)$

Type space: $T_1 = (t)$ [one type],

$T_2 = (t_1 : a = 2, t_2 : a = -2)$

Beliefs: $\mathbb{P}_1(a = 2) = \mathbb{P}_1(a = -2) = \frac{1}{2}$

$a = 2:$

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

$a = -2:$

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

Write as type-dependent payoff matrices

PS8, Ex. 2.b (A): Static Bayesian game (Bayesian Nash Equilibria)

(b) Now assume that player 2 knows the value of a, but player 1 only knows that $a = 2$ with probability 0.5 and $a = -2$ with probability 0.5. Explain how this situation can be modeled as a Bayesian game, describing the players, their action spaces, type spaces, beliefs and payoff functions.

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

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Type space: $T_1 = (t)$ [one type],

$T_2 = (t_1 : a = 2, t_2 : a = -2)$

Beliefs: $\mathbb{P}_1(a = 2) = \mathbb{P}_1(a = -2) = \frac{1}{2}$

Type-dependent payoff matrices:

Type $t_1 : a = 2$ ($p = \frac{1}{2}$)

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

Type $t_2 : a = -2$ ($p = \frac{1}{2}$)

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

(c) Find the Bayes-Nash equilibrium of the game described in (b).

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

(b) This can be modelled as a Bayesian game since P2 has two types (where he strongly prefers L and R respectively) and P1 has a belief about the distribution of these types (each happen $\frac{1}{2}$ of the time.).

Players: P1, P2

Action sp.: $A_1 = (U, D), A_2 = (L, R)$

Type space: $T_1 = (t)$ [one type],

$T_2 = (t_1 : a = 2, t_2 : a = -2)$

Beliefs: $\mathbb{P}_1(a = 2) = \mathbb{P}_1(a = -2) = \frac{1}{2}$

Type-dependent payoff matrices:

Type $t_1 : a = 2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

Type $t_2 : a = -2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

PS8, Ex. 2.c (A): Static Bayesian game (Bayesian Nash Equilibria)

(c) Find the Bayes-Nash equilibrium of the game described in (b).

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

(b) This can be modelled as a Bayesian game since P2 has two types (he either has L or R as a dominant strategy) and P1 has a belief about the distribution of these types (Each happen $\frac{1}{2}$ the time.).

Players: P1, P2

Action sp.: $A_1 = (U, D), A_2 = (L, R)$

Type space: $T_1 = (t)$ [one type],
$T_2 = (t_1 : a = 2, t_2 : a = -2)$

Beliefs: $\mathbb{P}_1(a = 2) = \mathbb{P}_1(a = -2) = \frac{1}{2}$

Type-dependent payoff matrices:

Type $t_1 : a = 2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

Type $t_2 : a = -2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

(c) The strategy for P1 is just one action, whereas the strategy for P2 is an action for each of the two types. Using this, we can write up the expected payoff matrix:

	LL	LR	RL	RR
U	$\frac{2}{2}+\frac{2}{2}, \frac{1}{2}+\frac{1}{2}$	$\frac{2}{2}+\frac{0}{2}, \frac{1}{2}-\frac{2}{2}$	$\frac{0}{2}+\frac{2}{2}, \frac{2}{2}+\frac{1}{2}$	$\frac{0}{2}+\frac{0}{2}, \frac{2}{2}-\frac{2}{2}$
D	$\frac{0}{2}+\frac{0}{2}, \frac{1}{2}+\frac{1}{2}$	$\frac{0}{2}+\frac{1}{2}, \frac{1}{2}-\frac{2}{2}$	$\frac{1}{2}+\frac{0}{2}, \frac{2}{2}+\frac{1}{2}$	$\frac{1}{2}+\frac{1}{2}, \frac{2}{2}-\frac{2}{2}$

PS8, Ex. 2.c (A): Static Bayesian game (Bayesian Nash Equilibria)

(c) Find the Bayes-Nash equilibrium of the game described in (b).

(a) The value of a not only affects P2's payoff, but also P2's strategy. For $a = 2$, P2 will have R as a dominant strategy; and for $a = -2$, P2 will have L as a dominant strategy.

(b) This can be modelled as a Bayesian game since P2 has two types (he either has L or R as a dominant strategy) and P1 has a belief about the distribution of these types (Each happen $\frac{1}{2}$ the time.).

Players: P1, P2

Action sp.: $A_1 = (U, D), A_2 = (L, R)$

Type space: $T_1 = (t)$ [one type],
$T_2 = (t_1 : a = 2, t_2 : a = -2)$

Beliefs: $\mathbb{P}_1(a = 2) = \mathbb{P}_1(a = -2) = \frac{1}{2}$

Type-dependent payoff matrices:

Type $t_1 : a = 2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

Type $t_2 : a = -2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

(c) The strategy for P1 is just one action, whereas the strategy for P2 is an action for each of the two types. Using this, we can write up the expected payoff matrix:

	LL	LR	RL	RR
U	2, 1	1,$-\frac{1}{2}$	1, $\frac{3}{2}$	0, 0
D	0, 1	$\frac{1}{2}$, $-\frac{1}{2}$	$\frac{1}{2}$, $\frac{3}{2}$	1, 0

Find the Bayesian Nash Equilibria.

PS8, Ex. 2.c (A): Static Bayesian game (Bayesian Nash Equilibria)

(c) Find the Bayes-Nash equilibrium of the game described in (b).

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Type-dependent payoff matrices:

Type $t_1 : a = 2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, 2
D	0, 1	1, 2

Type $t_2 : a = -2$ $(p = \frac{1}{2})$

	L	R
U	2, 1	0, -2
D	0, 1	1, -2

(c) The strategy for P1 is just one action, whereas the strategy for P2 is an action for each of the two types. Using this, we can write up the expected payoff matrix:

	LL	LR	RL	RR
U	2, 1	1, $-\frac{1}{2}$	1, $\frac{3}{2}$	0, 0
D	0, 1	$\frac{1}{2}$, $-\frac{1}{2}$	$\frac{1}{2}$, $\frac{3}{2}$	1, 0

The BNE is: (U,RL)

PS8, Ex. 3: Static Bayesian game (Bayesian Nash Equilibria)

PS8, Ex. 3: Static Bayesian game (Bayesian Nash Equilibria)

Exercise 3.4 in Gibbons (p. 169). Find all the pure-strategy Bayesian Nash equilibria in the following static Bayesian game:

a. Nature determines whether the payoffs are as in Game 1 or as in Game 2, each game being equally likely.
b. Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not.
c. Player 1 chooses either T or B; player 2 simultaneously chooses either L or R.
d. Payoffs are given by the game drawn by nature.

Game 1 $\left(t_1, p = \frac{1}{2}\right)$:

	L	R
T	1, 1	0, 0
B	0, 0	0, 0

Game 2 $\left(t_2, p = \frac{1}{2}\right)$:

	L	R
T	0, 0	0, 0
B	0, 0	2, 2

PS8, Ex. 3.a: Static Bayesian game (Bayesian Nash Equilibria)

Find all the Bayesian Nash equilibria in the following static Bayesian game:

Game 1 $\left(t_1, p = \frac{1}{2}\right)$:

	L	R
T	1, 1	0, 0
B	0, 0	0, 0

Game 2 $\left(t_2, p = \frac{1}{2}\right)$:

	L	R
T	0, 0	0, 0
B	0, 0	2, 2

Use the fact that each type of game happens half the time to write up the expected payoff matrix for all possible combinations of strategies:

- P2 plays L and P1 plays T if game is type 1 and T if game is type 2
- P2 plays L and P1 plays T if game is type 1 and B if game is type 2
- P2 plays L and P1 plays B if game is type 1 and T if game is type 2
- P2 plays L and P1 plays B if game is type 1 and B if game is type 2
- P2 plays R and P1 plays T if game is type 1 and T if game is type 2
- P2 plays R and P1 plays T if game is type 1 and B if game is type 2
- P2 plays R and P1 plays B if game is type 1 and T if game is type 2
- P2 plays R and P1 plays B if game is type 1 and B if game is type 2

PS8, Ex. 3.a: Static Bayesian game (Bayesian Nash Equilibria)

Find all the Bayesian Nash equilibria in the following static Bayesian game:

Game 1 $\left(t_1, p = \frac{1}{2}\right)$:

	L	R
T	1, 1	0, 0
B	0, 0	0, 0

Game 2 $\left(t_2, p = \frac{1}{2}\right)$:

	L	R
T	0, 0	0, 0
B	0, 0	2, 2

- P2 plays L and P1 plays T if game is type 1 and T if game is type 2
- P2 plays L and P1 plays T if game is type 1 and B if game is type 2
- P2 plays L and P1 plays B if game is type 1 and T if game is type 2
- P2 plays L and P1 plays B if game is type 1 and B if game is type 2
- P2 plays R and P1 plays T if game is type 1 and T if game is type 2
- P2 plays R and P1 plays T if game is type 1 and B if game is type 2
- P2 plays R and P1 plays B if game is type 1 and T if game is type 2
- P2 plays R and P1 plays B if game is type 1 and B if game is type 2

The expected payoff matrix:

Player 1 \ Player 2	L	R
TT	$\frac{1}{2}$, $\frac{1}{2}$	0, 0
TB	$\frac{1}{2}$, $\frac{1}{2}$	1, 1
BT	0, 0	0, 0
BB	0, 0	1, 1

Find all Bayesian Nash Equilibria.

PS8, Ex. 3.a: Static Bayesian game (Bayesian Nash Equilibria)

Find all the Bayesian Nash equilibria in the following static Bayesian game:

Game 1 $\left(t_1, p = \frac{1}{2}\right)$:

	L	R
T	1, 1	0, 0
B	0, 0	0, 0

Game 2 $\left(t_2, p = \frac{1}{2}\right)$:

	L	R
T	0, 0	0, 0
B	0, 0	2, 2

- P2 plays L and P1 plays T if game is type 1 and T if game is type 2
- P2 plays L and P1 plays T if game is type 1 and B if game is type 2
- P2 plays L and P1 plays B if game is type 1 and T if game is type 2
- P2 plays L and P1 plays B if game is type 1 and B if game is type 2
- P2 plays R and P1 plays T if game is type 1 and T if game is type 2
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- P2 plays R and P1 plays B if game is type 1 and B if game is type 2

The expected payoff matrix:

Player 2

Player 1	L	R
TT	$\frac{1}{2}$, $\frac{1}{2}$	0, 0
TB	$\frac{1}{2}$, $\frac{1}{2}$	1, 1
BT	0, 0	0, 0
BB	0, 0	1, 1

This gives the following BNE:

$$BNE = \{(TT, L), (TB, R), (BB, R)\}$$

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

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1. The timing is as follows where p is a commonly known distribution:
    1.1 Nature draws all players' type according to p.
    1.2 Each player i learns her own type t_i.
    1.3 Players form their beliefs about the type profile.
    1.4 Players simultaneously choose actions and payoffs are realized.

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

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   1.3 Players form their beliefs about the type profile.
   1.4 Players simultaneously choose actions and payoffs are realized.
2. The static Bayesian game consists of:
   2.1 Players: $Player\ 1, ..., Player\ n$
   2.2 Type spaces: $T_1 = \{t_{11}, ..., t_{1K}\}, ...$
   2.3 Beliefs: $\mathbb{P}_1[t_2 = t_{21}] = \cdot, ...$
   2.4 Action spaces: $A_1 = \{\cdot\}, ...$
   2.5 Strategy spaces: $S_1 = \{s_1(t_1), \cdot\} = \{(s_1|t_{11}, ..., s_1|t_{1K}), \cdot\}, ...$
   2.6 Type-dependent payoff matrices.

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

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   2.4 Action spaces: $A_1 = \{\cdot\}, ...$
   2.5 Strategy spaces: $S_1 = \{s_1(t_1), \cdot\} = \{(s_1|t_{11}, ..., s_1|t_{1K}), \cdot\}, ...$
   2.6 Type-dependent payoff matrices.
3. Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for a player i (the player with the smallest strategy space). For each strategy $s_i(t_i)$:
   3.1 Write up the best response of the other player(s): $s_j^*(t_j) \equiv BR_j\left(s_i(t_i)|t_j\right)$.
   3.2 If $s_i(t_i) = BR_i\left(s_j(t_j)|t_i\right) \equiv s_i^*(t_i)$ then $\left(s_i^*(t_i), s_j^*(t_j)\right)$ is a BNE.

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

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   1.1 Nature draws all players' type according to p.
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   3.1 Write up the best response of the other player(s): $s_j^*(t_j) \equiv BR_j\left(s_i(t_i)|t_j\right)$.
   3.2 If $s_i(t_i) = BR_i\left(s_j(t_j)|t_i\right) \equiv s_i^*(t_i)$ then $\left(s_i^*(t_i), s_j^*(t_j)\right)$ is a BNE.
4. In BNE, a strategy must maximize expected utility given the strategy of the other player(s) and the probability of them being each type, i.e. no type of any player has an incentive to deviate as in equilibrium player i's strategy is a best response to player j's strategy given player i's beliefs:

$$\max_{s_i} \sum_{j \neq i} \sum_{t_{jk} \in T_j} \mathbb{P}_i[t_j = t_{jk}] \cdot u_i\left(s_i(t_i), s_j^*(t_j)|t_i\right)$$

PS8, Ex. 4: The dating game (Bayesian Nash Equilibria)

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Consider the 'dating game'. In this game, player 1 wants to meet with player 2, even if player 2 is of type t_2, who does not want to meet with player 1. Player 2's types have equal probability.

Now suppose we construct an alternative version of the game, where player 1 has self respect, and only wants to meet up with player 2 if player 2 also wants to meet up (i.e. if player 2 is of type t_1). Otherwise, player 1 prefers to *miscoordinate* (i.e. not to meet up) as well. Player 1 still has the same preference for going to football versus the opera.

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2's type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

(b) Find the set of (pure strategy) Bayesian Nash Equilibria of this game.

[Hints on the next slide.]

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Consider the 'dating game'. In this game, player 1 wants to meet with player 2, even if player 2 is of type t_2, who does not want to meet with player 1. Player 2's types have equal probability.

Now suppose we construct an alternative version of the game, where player 1 has self respect, and only wants to meet up with player 2 if player 2 also wants to meet up (i.e. if player 2 is of type t_1). Otherwise, player 1 prefers to *miscoordinate* (i.e. not to meet up) as well. Player 1 still has the same preference for going to football versus the opera.

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2's type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

Hint: Write up the Bayesian game (players, type spaces, beliefs, action spaces, strategy spaces, and the type-dependent payoff matrices.)

(b) Find the set of (pure strategy) Bayesian Nash Equilibria of this game.

Hints:

1. Check for equilibria where player 1 plays *Football* and *Opera* respectively.
2. In equilibrium, a strategy should maximize expected payoff given the strategy of the other player and the probability of each type.

PS8, Ex. 4.a: The dating game (Bayesian Nash Equilibria)

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2's type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

1. Players: P1, P2.
2. Type spaces: $T_1 = \{t\},\ T_2 = \{t_1, t_2\}$
3. Beliefs: $\mathbb{P}_1(T_2 = t_1) = \mathbb{P}_1(T_2 = t_2) = \frac{1}{2},\ \mathbb{P}_2(T_1 = t) = 1$
4. Action space: $A_i = \{Football, Opera\}$, for $i \in 1, 2$
5. Strategy spaces: $S_1 = \{F, O\},\ S_2 = \{FF, FO, OF, OO\}$
6. Type-dependent payoff matrices:

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*.

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.a: $BR_2(F) = (FO)$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.a: $BR_2(F) = (FO)$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$
$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$
$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$

$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$
$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$

$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$

$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2 \text{ plays } OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$

$u_1(F|P2 \text{ plays } OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$
$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2 \text{ plays } OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$
$u_1(F|P2 \text{ plays } OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$
$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e.
$BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2\ plays\ OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$
$u_1(F|P2\ plays\ OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e.
$BNE_2 = \{O, OF\}$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

Step 3: Write up the set of all BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$
$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e.
$BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2 \text{ plays } OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$
$u_1(F|P2 \text{ plays } OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e.
$BNE_2 = \{O, OF\}$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

Step 3: Write up the set of all BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$

$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2 \text{ plays } OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$

$u_1(F|P2 \text{ plays } OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e. $BNE_2 = \{O, OF\}$

3: $BNE = \{(F, FO), (O, OF)\}$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

Step 3: Write up the set of all BNE.

Alternative: Instead, do as in ex. 3 and 4 (less elegant as you also need to calculate expected payoffs that are irrelevant).

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2\ plays\ OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$

$u_1(F|P2\ plays\ OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e. $BNE_2 = \{O, OF\}$

3: $BNE = \{(F, FO), (O, OF)\}$

PS8, Ex. 4.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

The expected payoff matrix:

	FF	FO	OF	OO
F	$1, \frac{1}{2}$	$2, \frac{3}{2}$	0, 0	1, 1
O	$\frac{1}{2}, \frac{1}{2}$	0, 0	$1, \frac{3}{2}$	$\frac{1}{2}$, 1

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

Step 3: Write up the set of all BNE.

Alternative: Instead, do as in ex. 3 and 4 (less elegant as you also need to calculate expected payoffs that are irrelevant).

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$
$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2\ plays\ OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$
$u_1(F|P2\ plays\ OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e. $BNE_2 = \{O, OF\}$

3: $BNE = \{(F, FO), (O, OF)\}$